INVERSE TRIGONOMETRICE FUNCTIONS

1. If $[\sin^{-1}\cos^{-1}\sin^{-1}x]=1$, where [.] denotes the greatest integer function, then x belongs to the interval
   a) [tan sin cos 1, tan sin cos sin 1] b) (tan sin cos 1, tan sin cos sin 1)
   c) $[-1,1]$ d) [sin cos tan 1, sin cos sin tan 1]
2. $\sec^2(\tan^{-1}2)+\operatorname{cosec}^2(\cot^{-1}3)$ is equal to
   a) 1 b) 5 c) 10 d) 15
3. If $2\tan^{-1}(\cos x)=\tan^{-1}(2\operatorname{cosec} x)$, then the value of x is
   a) $\frac{3\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) None of these
4. If $\tan^{-1}\frac{x-1}{x+2}+\tan^{-1}\frac{x+1}{x+2}=\frac{\pi}{4}$, then x is equal to
   a) $\frac{1}{\sqrt{2}}$ b) $-\frac{1}{\sqrt{2}}$ c) $\pm\sqrt{\frac{5}{2}}$ d) $\pm\frac{1}{2}$
5. $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}}$ is equal to
   a) $2\sin^{-1}\frac{x}{a}$ b) $\sin^{-1}\frac{2x}{a}$ c) $\sin^{-1}\frac{x}{a}$ d) $\cos^{-1}\frac{x}{a}$
6. The sum of the infinite series
   $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\sin^{-1}\left(\frac{\sqrt{2}-1}{\sqrt{6}}\right)+\sin^{-1}\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{12}}\right)+\dots$
   $+\dots+\sin^{-1}\left(\frac{\sqrt{n}-\sqrt{(n-1)}}{\sqrt{\{n(n+1)\}}}\right)+\dots$ is
   a) $\frac{\pi}{8}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) π
7. If $\theta_1=\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{1}{3}$ and $\theta_2=\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{1}{3}$, then
   a) $\theta_1>\theta_2$ b) $\theta_1=\theta_2$ c) $\theta_1<\theta_2$ d) None of these
8. If $\cos^{-1}x>\sin^{-1}x$, then
   a) $x<0$ b) $-1<x<0$ c) $0\le x<\frac{1}{\sqrt{2}}$ d) $-1\le x<\frac{1}{\sqrt{2}}$
9. If $e^{[\sin^2\alpha+\sin^4\alpha+\sin^6\alpha+\dots\infty]\log_e 2}$ is a root of equation $x^2-9x+8=0$,where $0<\alpha<\frac{\pi}{2}$, then the principle value of $\sin^{-1}\sin\left(\frac{2\pi}{3}\right)$ is
   a) α b) 2α c) $-\alpha$ d) -2α
10. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z+\cos^{-1}t=4\pi$, then the value of $x^2+y^2+z^2+t^2$ is
   a) $xy+zy+zt$ b) $1-2xyzt$ c) 4 d) 6
11. Sum of infinite terms of the series
   $\cot^{-1}\left(1^2+\frac{3}{4}\right)+\cot^{-1}\left(2^2+\frac{3}{4}\right)+\cot^{-1}\left(3^2+\frac{3}{4}\right)+\dots$ is
   a) $\frac{\pi}{4}$ b) $\tan^{-1}(2)$ c) $\tan^{-1}3$ d) None of these
12. If $\sin^{-1}\alpha+\sin^{-1}\beta+\sin^{-1}\gamma=\frac{3\pi}{2}$, then $\alpha\beta+\alpha\gamma+\beta\gamma$ is equal to
   a) 1 b) 0 c) 3 d) -3
13. The value of $\tan^{-1}(1)+\tan^{-1}(0)+\tan^{-1}(2)+\tan^{-1}(3)$ is equal to

a) π b) $\frac{5\pi}{4}$ c) $\frac{\pi}{2}$ d) None of these

14. If $\cos^{-1}p + \cos^{-1}q + \cos^{-1}r = 3\pi$, then $p^2 + q^2 + r^2 + 2pqr$ is equal to
a) 3 b) 1 c) 2 d) -1

15. If $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x \geq 0$, then the smallest interval in which θ lies, is given by
a) $\frac{\pi}{2} \leq \theta \leq \frac{3\pi}{4}$ b) $-\frac{\pi}{4} \leq \theta \leq 0$ c) $0 \leq \theta \leq \frac{\pi}{4}$ d) $\frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$

16. $\cot\left\{\cos^{-1}\left(\frac{7}{25}\right)\right\} =$
a) $\frac{25}{24}$ b) $\frac{25}{7}$ c) $\frac{24}{25}$ d) None of these

17. If $x \in (1, \infty)$, then $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ equals
a) $2\tan^{-1}x$ b) $\pi - 2\tan^{-1}x$ c) $-\pi - 2\tan^{-1}x$ d) None of these

18. Cos $[\tan^{-1}\{\sin(\cot^{-1}x)\}]$ is equal to
a) $\sqrt{\frac{x^2+2}{x^2+3}}$ b) $\sqrt{\frac{x^2+2}{x^2+1}}$ c) $\sqrt{\frac{x^2+1}{x^2+2}}$ d) None of these

19. The value of $\tan\left\{\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right\}$is
a) $\frac{2}{3\sqrt{5}}$ b) $\frac{2}{3}$ c) $\frac{1}{\sqrt{5}}$ d) $\frac{4}{\sqrt{5}}$

20. The solution set of the equation $\tan^{-1}x - \cot^{-1}x = \cos^{-1}(2 - x)$is
a) $[0,1]$ b) $[-1,1]$ c) $[1,3]$ d) None of these

21. The value of $\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$ is equal to
a) π b) $\frac{5\pi}{4}$ c) $\frac{\pi}{2}$ d) None of these

22. If a, b are positive quantities and if $a_1 = \frac{a+b}{2}, b_1 = \sqrt{a_1 b}, a_2 = \frac{a_1+b_1}{2}, b_2 = \sqrt{a_2 b_1}$ and so on, then
a) $a_\infty = \frac{\sqrt{b^2-a^2}}{\cos^{-1}\left(\frac{a}{b}\right)}$ b) $b_\infty = \frac{\sqrt{b^2-a^2}}{\cos^{-1}\left(\frac{a}{b}\right)}$ c) $b_\infty = \frac{\sqrt{a^2+b^2}}{\cos^{-1}\left(\frac{b}{a}\right)}$ d) None of these

23. The value of $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is
a) $\frac{\sqrt{3}}{2}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{1}{2}$ d) $-\frac{1}{2}$

24. The value of $\sin[\cot^{-1}\{\cos(\tan^{-1}x)\}]$, is
a) $\sqrt{\frac{x^2+2}{x^2+1}}$ b) $\sqrt{\frac{x^2+1}{x^2+2}}$ c) $\frac{x}{\sqrt{x^2+2}}$ d) $\frac{1}{\sqrt{x^2+2}}$

25. $\cot^{-1}(2.1^2) + \cot^{-1}(2.2^2) + \cot^{-1}(2.3^2)+\ldots$ upto ∞ is equal to
a) $\frac{\pi}{4}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{2}$ d) $\frac{\pi}{5}$

26. If $x \in (1, \infty)$, then $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ equals
a) $2\tan^{-1}x$ b) $-\pi + 2\tan^{-1}x$ c) $\pi + 2\tan^{-1}x$ d) None of these

27. The value of $\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$ is
a) 1 b) 3 c) 0 d) $-\frac{2\sqrt{6}}{5}$

28. The value of $\cot^{-1}\frac{xy+1}{x-y} + \cot^{-1}\frac{yz+1}{y-z} + \cot^{-1}\frac{zx+1}{z-x}$ is
a) 0 b) 1
c) $\cot^{-1}x + \cot^{-1}y + \cot^{-1}z$ d) None of the above

29. If we consider only the principle value of the inverse trigonometric functions, then the value of $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right)$ is

a) $\sqrt{\frac{29}{3}}$ b) $\frac{29}{3}$ c) $\sqrt{\frac{3}{29}}$ d) $\frac{3}{29}$

30. The numerical value of $\tan\left(2\tan^{-1}\frac{1}{5}-\frac{\pi}{4}\right)$, is

a) 1 b) 0 c) $\frac{7}{17}$ d) $-\frac{7}{17}$

31. If in a $\Delta ABC, \angle A=\tan^{-1}2$ and $\angle B=\tan^{-1}3$, then angle C is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) None of these

32. $\cos^{-1}\left\{\frac{1}{2}x^2+\sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right\}=\cos^{-1}\frac{x}{2}-\cos^{-1}x$ holds for

a) $|x|\le 1$ b) $x\in R$ c) $0\le x\le 1$ d) $-1\le x\le 0$

33. If θ and ϕ are the roots of the equation $8x^2+22x+5=0$, then

a) Both $\sin^{-1}\theta$ and $\sin^{-1}\phi$ are equal b) Both $\sec^{-1}\theta$ and $\sec^{-1}\phi$ are equal
c) Both $\tan^{-1}\theta$ and $\tan^{-1}\phi$ are equal d) None of the above

34. If $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$, then x is equal to

a) 0 b) 2 c) 1 d) -1

35. $\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$ is equal to

a) $\tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$ b) $\tan^{-1}\left(\frac{n^2-n}{n^2-n+2}\right)$ c) $\tan^{-1}\left(\frac{n^2+n+2}{n^2+n}\right)$ d) None of these

36. If we consider only the principle value of the inverse trigonometric functions, then the value of $\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}}-\sin^{-1}\frac{4}{\sqrt{17}}\right)$ is

a) $\sqrt{\frac{29}{3}}$ b) $\frac{29}{3}$ c) $\sqrt{\frac{3}{29}}$ d) $\frac{3}{29}$

37. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$, then the value of $x^{100}+y^{100}+z^{100}-\frac{9}{x^{101}+y^{101}+z^{101}}$ is

a) 0 b) 1 c) 2 d) 3

38. If $y=\cos^{-1}(\cos 10)$, then y is equal to

a) 10 b) $4\pi-10$ c) $2\pi+10$ d) $2\pi-10$

39. If $\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}=\tan^{-1}\frac{2x}{1-x^2}$, then value of x is

a) a b) b c) $\frac{a+b}{1-ab}$ d) $\frac{a-b}{1+ab}$

40. The value of $\sum_{r=0}^{\infty}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{4}$ d) None of these

41. $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$ is equal to

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

42. $\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}+\tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}}+\tan^{-1}\sqrt{\frac{c(a+b+c)}{a\,b}}$, is

a) $\pi/4$ b) $\pi/2$ c) π d) 0

43. If $\sec^{-1}\sqrt{1+x^2}+\text{cosec}^{-1}\frac{\sqrt{1+y^2}}{y}+\cot^{-1}\frac{1}{z}=\pi$, then $x+y+z$ is equal to

a) xyz b) $2xyz$ c) xyz^2 d) x^2yz

44. If $\tan^{-1}(x-1)+\tan^{-1}x+\tan^{-1}(x+1)=\tan^{-1}3x$, then x is

a) $\pm\frac{1}{2}$ b) $0, \frac{1}{2}$ c) $0, -\frac{1}{2}$ d) $0, \pm\frac{1}{2}$

45. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$, then the value of $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is

a) 0 b) 1 c) 2 d) 3

46. $\sin^{-1}x + \sin^{-1}\frac{1}{x} + \cos^{-1}x + \cos^{-1}\frac{1}{x}$ is equal to

a) π b) $\frac{\pi}{2}$ c) $\frac{3\pi}{2}$ d) None of these

47. $\sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$ is equal to

a) $\tan^{-1}\left(\frac{n^2+n}{n^4+n^2+2}\right)$ b) $\tan^{-1}\left(\frac{n^2-n}{n^2-n+2}\right)$ c) $\tan^{-1}(n^2+n+2)$ d) None of these

48. If $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$, then the value of $a\sqrt{(1-a^2)} + b\sqrt{(1-b^2)} + c\sqrt{1-c^2)}$ will be

a) $2abc$ b) abc c) $\frac{1}{2}abc$ d) $\frac{1}{3}abc$

49. Which one of the following is correct?

a) $\tan 1 > \tan^{-1} 1$ b) $\tan 1 < \tan^{-1} 1$ c) $\tan 1 = \tan^{-1} 1$ d) None of these

50. The value of $\cos(2\cos^{-1} 0.8)$ is

a) 0.48 b) 0.96 c) 0.6 d) None of these

51. The solution of $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ is

a) $\frac{1}{6}$ b) -1 c) $\left(\frac{1}{6}, -1\right)$ d) None of these

52. The value of $\cos\left[\frac{1}{2}\cos^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{63}}{8}\right)\right\}\right]$, is

a) $\frac{3}{16}$ b) $\frac{3}{8}$ c) $\frac{3}{4}$ d) $\frac{3}{2}$

53. If $0 \le x \le 1$, then $\cos^{-1}(2x^2-1)$ equals

a) $2\cos^{-1}x$ b) $\pi - 2\cos^{-1}x$ c) $2\pi - 2\cos^{-1}x$ d) None of these

54. The value of $\sin(\cot^{-1}x)$ is

a) $\sqrt{1+x^2}$ b) x c) $(1+x^2)^{-3/2}$ d) $(1+x^2)^{-1/2}$

55. Value of $\tan^{-1}\left(\frac{\sin 2 - 1}{\cos 2}\right)$ is

a) $\frac{\pi}{2} - 1$ b) $1 - \frac{\pi}{4}$ c) $2 - \frac{\pi}{2}$ d) $\frac{\pi}{4} - 1$

56. If $\angle A = 90°$ in the triangle ABC, then $\tan^{-1}\left(\frac{c}{a+b}\right) + \tan^{-1}\left(\frac{b}{a+c}\right)$ is equal to

a) 0 b) 1 c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

57. If $-\frac{1}{2} \le x \le \frac{1}{2}$, then $\cos^{-1}(4x^3 - 3x)$ equals

a) $3\cos^{-1}x$ b) $2\pi - 3\cos^{-1}x$ c) $-2\pi - 3\cos^{-1}x$ d) None of these

58. If $\sin^{-1}a + \sin^{-1}b + \sin^{-1}c = \pi$, then the value of $a\sqrt{(1-a^2)} + b\sqrt{(1-b^2)} + c\sqrt{1-c^2)}$ will be

a) $2abc$ b) abc c) $\frac{1}{2}abc$ d) $\frac{1}{3}abc$

59. If $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$, then x belongs to

a) {1, 0} b) {-1, 1} c) $\left\{0, \frac{1}{2}\right\}$ d) {2, 0}

60. If $\tan^{-1}a + \tan^{-1}b = \sin^{-1}1 - \tan^{-1}c$, then

a) $a + b + c = abc$ b) $ab + bc + ca = abc$

c) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} - \frac{1}{abc} = 0$ d) $ab + bc + ca = a + b + c$

61. The value of x for which $\cos^{-1}(\cos 4) > 3x^2 - 4x$ is

a) $\left(0, \frac{2+\sqrt{6\pi-8}}{3}\right)$ b) $\left(\frac{2-\sqrt{6\pi-8}}{3}, 0\right)$

c) $(-2,2)$ d) $\left(\frac{2-\sqrt{6\pi-8}}{3}, \frac{2+\sqrt{6\pi-8}}{3}\right)$

62. If x takes negative permissible value, then $\sin^{-1} x$ is equal to

a) $-\cos^{-1}\sqrt{1-x^2}$ b) $\cos^{-1}\sqrt{x^2-1}$ c) $\pi-\cos^{-1}\sqrt{1-x^2}$ d) $\cos^{-1}\sqrt{1-x^2}$

63. The value of $\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{7}{8}$ is

a) $\tan^{-1}\frac{7}{8}$ b) $\cot^{-1} 15$ c) $\tan^{-1} 15$ d) $\tan^{-1}\frac{25}{24}$

64. The smallest and the largest values of $\tan^{-1}\left(\frac{1-x}{1+x}\right), 0 \le x \le 1$ are

a) $0, \pi$ b) $0, \frac{\pi}{4}$ c) $-\frac{\pi}{4}, \frac{\pi}{4}$ d) $\frac{\pi}{4}, \frac{\pi}{2}$

65. If $\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}=\tan^{-1}\frac{2x}{1-x^2}$, then the value of x is

a) a b) b c) $\frac{a+b}{1-ab}$ d) $\frac{a-b}{1+ab}$

66. Number of solutions of the equation
$\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$ is

a) 1 b) 2 c) 3 d) 4

67. If $-\frac{1}{2} \le x \le \frac{1}{2}$, then $\sin^{-1}(3x-4x^3)$ equals

a) $3\sin^{-1} x$ b) $\pi-3\sin^{-1} x$ c) $-\pi-3\sin^{-1} x$ d) None of these

68. If $x \in (-\infty, -1)$, then $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ equals

a) $2\tan^{-1} x$ b) $\pi-2\tan^{-1} x$ c) $-\pi-2\tan^{-1} x$ d) None of these

69. The sum of the infinite series

$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)+\sin^{-1}\left(\frac{\sqrt{2}-1}{\sqrt{6}}\right)+\sin^{-1}\left(\frac{\sqrt{3}-\sqrt{2}}{\sqrt{12}}\right)+\dots$

$+\dots+\sin^{-1}\left(\frac{\sqrt{n}-\sqrt{(n-1)}}{\sqrt{\{n(n+1)\}}}\right)+\dots$ is

a) $\frac{\pi}{8}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) π

70. If $\cos^{-1} x=\alpha, (0<x<1)$ and
$\sin^{-1}(2x\sqrt{1-x^2})+\sec^{-1}\left(\frac{1}{2x^2-1}\right)=\frac{2\pi}{3}$, then $\tan^{-1}(2x)$ equals

a) $\pi/6$ b) $\pi/4$ c) $\pi/3$ d) $\pi/2$

71. If $\alpha=\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{1}{3}$ and $\beta=\cos^{-1}\frac{4}{5}+\cos^{-1}\frac{1}{3}$, then

a) $\alpha<\beta$ b) $\alpha=\beta$ c) $\alpha>\beta$ d) None of these

72. If $\theta=\tan^{-1} a, \phi=\tan^{-1} b$ and $ab=-1$, then $(\theta-\phi)$ is equal to

a) 0 b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) None of these

73. If $\tan\theta+\tan\left(\frac{\pi}{3}+\theta\right)+\tan\left(-\frac{\pi}{3}+\theta\right)=a\tan 3\theta$, then a is equal to

a) 1/3 b) 1 c) 3 d) None of these

74. If the $(\cos^{-1} x)=\sin\left(\cot^{-1}\frac{1}{2}\right)$, then x is equal to

a) $\pm\frac{5}{3}$ b) $\pm\frac{\sqrt{5}}{3}$ c) $\pm\frac{5}{\sqrt{3}}$ d) None of these

75. If $\sin^{-1}=\frac{\pi}{5}$, for some $x \in (-1,1)$, then the value of $\cos^{-1} x$ is

a) $\frac{3\pi}{10}$ b) $\frac{5\pi}{10}$ c) $\frac{7\pi}{10}$ d) $\frac{9\pi}{10}$

76. If $\frac{1}{2} \leq x \leq 1$, then $\sin^{-1}(3x - 4x^3)$ equals

a) $3\sin^{-1}x$
b) $\pi - 3\sin^{-1}x$
c) $-\pi - 3\sin^{-1}x$
d) None of these

77. $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} =$

a) $\frac{\pi}{3}$
b) $\frac{\pi}{4}$
c) $\frac{\pi}{2}$
d) 0

78. If x, y, z are in AP and $\tan^{-1}x, \tan^{-1}y$ and $\tan^{-1}z$ are also in AP, then

a) $x = y = z$
b) $x = y = -z$
c) $x = 1, y = 2, z = 3$
d) $x = 2, y = 4, z = 6$

79. If $a_1, a_2, a_3, \dots a_n$ are in AP with common difference 5 and if $a_i a_j \neq -1$ for $i, j = 1, 2, \dots, n$ then $\tan^{-1}\left(\frac{5}{1+a_1a_2}\right) + \tan^{-1}\left(\frac{5}{1+a_2a_3}\right) + \dots + \tan^{-1}\left(\frac{5}{1+a_{n-1}a_n}\right)$ is equal to

a) $\tan^{-1}\left(\frac{5}{1+a_na_{n-1}}\right)$
b) $\tan^{-1}\left(\frac{5a_1}{1+a_na_1}\right)$
c) $\tan^{-1}\left(\frac{5n-5}{1+a_na_1}\right)$
d) $\tan^{-1}\left(\frac{5n-5}{1+a_1a_{n+1}}\right)$

80. The relation $\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \tan^{-1}x$ holds true for all

a) $x \in R$
b) $x \in (-\infty, 1)$
c) $x \in (-1, \infty)$
d) $x \in (-\infty, -1)$

81. If $A = \tan^{-1}\left(\frac{x\sqrt{3}}{2k-x}\right)$ and $B = \tan^{-1}\left(\frac{2x-k}{k\sqrt{3}}\right)$, then the value of $A - B$ is

a) $10°$
b) $45°$
c) $60°$
d) $30°$

82. If $0 < x < 1$, then $\sqrt{1+x^2}[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2}$ is equal to

a) $\frac{x}{\sqrt{1+x^2}}$
b) x
c) $x\sqrt{1+x^2}$
d) $\sqrt{1+x^2}$

83. If $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$, then value of x is

a) 1
b) 3
c) 4
d) 5

84. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$ and $f(1) = 2$, $f(p+q) = f(p).f(q), \forall\, p, q \in R$, then $x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{(x+y+z)}{x^{f(1)}+y^{f(2)}+z^{f(3)}}$ is equal to

a) 0
b) 1
c) 2
d) 3

85. If $A = 2\tan^{-1}(2\sqrt{2} - 1)$ and $B = 3\sin^{-1}\frac{1}{3} + \sin^{-1}\frac{3}{5}$, then

a) $A = B$
b) $A < B$
c) $A > B$
d) None of these

86. If $\tan^{-1}(x+2) + \tan^{-1}(x-2) - \tan^{-1}\left(\frac{1}{2}\right) = 0$, then one of the values of x is equal to

a) -1
b) 5
c) $\frac{1}{2}$
d) 1

87. $\cos\left[\cos^{-1}\left(-\frac{1}{7}\right) + \sin^{-1}\left(-\frac{1}{7}\right)\right]$ is equal to

a) $-\frac{1}{3}$
b) 0
c) $\frac{1}{3}$
d) $\frac{4}{9}$

88. If $\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$, then x is

a) $\frac{1}{2}$
b) $\frac{\sqrt{3}}{2}$
c) $-\frac{1}{2}$
d) None of these

89. The value of $\sec\left[\tan^{-1}\left(\frac{b+a}{b-a}\right) - \tan^{-1}\left(\frac{a}{b}\right)\right]$ is

a) 2
b) $\sqrt{2}$
c) 4
d) 1

90. Which one of following is true?

a) $\sin(\cos^{-1}x) = \cos(\sin^{-1}x)$
b) $\sec(\tan^{-1}x) = \tan(\sec^{-1}x)$
c) $\cos(\tan^{-1}x) = \tan(\cos^{-1}x)$
d) $\tan(\sin^{-1}x) = \sin(\tan^{-1}x)$

91. If $a > b > 0$, then the value of $\tan^{-1}\left(\frac{a}{b}\right) + \tan^{-1}\left(\frac{a+b}{a-b}\right)$ depends on

a) Both a and b
b) b and not a
c) a and not b
d) Neither a nor b

92. If $x \geq 1$, then $2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is equal to

a) $4\tan^{-1}x$ b) 0 c) $\pi/2$ d) π

93. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, then

a) $x^2 + y^2 = z^2$ b) $x^2 + y^2 + z^2 = 0$
c) $x^2 + y^2 + z^2 = 1 - 2xyz$ d) None of the above

94. If $x > \frac{1}{\sqrt{3}}$, then $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ equals

a) $3\tan^{-1}x$ b) $-\pi + 3\tan^{-1}x$ c) $\pi + 3\tan^{-1}x$ d) None of these

95. If $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$, then the value of x is

a) $\frac{3\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) None of these

96. The value of $\cot^{-1}\frac{3}{4} + \sin^{-1}\frac{5}{13}$ is

a) $\sin^{-1}\frac{63}{65}$ b) $\sin^{-1}\frac{12}{13}$ c) $\sin^{-1}\frac{65}{68}$ d) $\sin^{-1}\frac{5}{12}$

97. If $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$ and $\tan^{-1}x - \tan^{-1}y = 0$, then $x^2 + xy + y^2$ is equal to

a) 0 b) $\frac{1}{\sqrt{2}}$ c) $\frac{3}{2}$ d) $\frac{1}{8}$

98. The number of real solution of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is

a) 0 b) 1 c) 2 d) ∞

99. If $x + y + z = xyz$, then $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z =$

a) 0 b) $\pi/2$ c) 1 d) None of these

100. The number of positive integral solutions of the equation $\tan^{-1}x + \cos^{-1}\frac{y}{\sqrt{1+y^2}} = \sin^{-1}\frac{3}{\sqrt{10}}$ is

a) One b) Two c) Zero d) None of these

101. $\sin^{-1}\left(\frac{3}{5}\right) + \tan^{-1}\left(\frac{1}{7}\right) =$

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\cos^{-1}\left(\frac{4}{5}\right)$ d) π

102. If $xy + yz + zx = 1$, then $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z =$

a) π b) $\pi/2$ c) 1 d) none of these

103. If $x^2 + y^2 + z^2 = r^2$, then
$\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{xz}{yr}\right)$ is equal to

a) π b) $\frac{\pi}{2}$ c) 0 d) None of these

104. If $f(x) = \sin^{-1}\left\{\frac{\sqrt{3}}{2}x - \frac{1}{2}\sqrt{1-x^2}\right\}, -\frac{1}{2} \leq x \leq 1$, then $f(x)$ is equal to

a) $\sin^{-1}\frac{1}{2} - \sin^{-1}x$ b) $\sin^{-1}x - \frac{\pi}{6}$ c) $\sin^{-1}x + \frac{\pi}{6}$ d) None of these

105. $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$ is equal to

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{2\pi}{3}$ d) $\frac{\pi}{4}$

106. The solution of $\tan^{-1}2\theta + \tan^{-1}3\theta = \frac{\pi}{4}$ is

a) $\frac{1}{\sqrt{3}}$ b) $\frac{1}{3}$ c) $\frac{1}{6}$ d) $\frac{1}{\sqrt{6}}$

107. The value of $\cos^{-1}\left(-\frac{1}{2}\right)$ among the following, is

a) $\frac{9\pi}{3}$ b) $\frac{8\pi}{3}$ c) $\frac{5\pi}{3}$ d) $\frac{11\pi}{3}$

108. If $\tan\theta + \tan\left(\frac{\pi}{3} + \theta\right) + \tan\left(-\frac{\pi}{3} + \theta\right) = a\tan 3\theta$, then a is equal to

a) 1/3 b) 1 c) 3 d) None of these

109. The value of $\cot^{-1}\frac{3}{4}+\sin^{-1}\frac{5}{13}$ is

a) $\sin^{-1}\frac{63}{65}$ b) $\sin^{-1}\frac{12}{13}$ c) $\sin^{-1}\frac{65}{68}$ d) $\sin^{-1}\frac{5}{12}$

110. The value of x for which $\cos^{-1}(\cos 4) > 3x^2-4x$ is

a) $\left(0,\frac{2+\sqrt{6\pi-8}}{3}\right)$ b) $\left(\frac{2-\sqrt{6\pi-8}}{3},0\right)$

c) $(-2,2)$ d) $\left(\frac{2-\sqrt{6\pi-8}}{3},\frac{2+\sqrt{6\pi-8}}{3}\right)$

111. If $x\in(-\infty,1)$, then $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ equals

a) $2\tan^{-1}x$ b) $-\pi+2\tan^{-1}x$ c) $\pi+2\tan^{-1}x$ d) None of these

112. If $\frac{1}{\sqrt{2}}\le x\le 1$, then $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ equals

a) $2\sin^{-1}x$ b) $\pi-2\sin^{-1}x$ c) $-\pi-2\sin^{-1}x$ d) None of these

113. $\frac{\alpha^3}{2}\operatorname{cosec}^2\left(\frac{1}{2}\tan^{-1}\ \frac{\alpha}{\beta}\right)+\frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\ \left(\frac{\beta}{\alpha}\right)\right)$ is

a) $(\alpha-\beta)(\alpha^2+\beta^2)$ b) $(\alpha+\beta)(\alpha^2-\beta^2)$

c) $(\alpha+\beta)(\alpha^2+\beta^2)$ d) None of these

114. If $\sum_{i=1}^{20}\sin^{-1}x_i=10\pi$,then $\sum_{i=1}^{20}x_i$ is equal to

a) 20 b) 10 c) 0 d) None of these

115. Which one of the following is correct?

a) $\tan 1>\tan^{-1}1$ b) $\tan 1<\tan^{-1}1$ c) $\tan 1=\tan^{-1}1$ d) None of these

116. If $\alpha=\sin^{-1}\frac{\sqrt{3}}{2}+\sin^{-1}\frac{1}{3}$ and $\beta=\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\frac{1}{3}$, then

a) $\alpha>\beta$ b) $\alpha=\beta$ c) $\alpha<\beta$ d) $\alpha+\beta=2\pi$

117. $2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)$is equal to

a) $\left(\frac{49}{29}\right)$ b) $\frac{\pi}{2}$ c) $-\left(\frac{49}{29}\right)$ d) $\frac{\pi}{4}$

118. $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right)+\frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$ is equal to

a) $\frac{2a}{1+a^2}$ b) $\frac{1-a^2}{1+a^2}$ c) $\frac{2a}{1-a^2}$ d) None of these

119. The sum of the infinite series

$\cot^{-1}2+\cot^{-1}8+\cot^{-1}18+\cot^{-1}32+\ldots$ is

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) None of these

120. If $\tan^{-1}\left(\frac{a}{x}\right)+\tan^{-1}\left(\frac{b}{x}\right)=\frac{\pi}{2}$, then x is equal to

a) $\sqrt{ab}$ b) $\sqrt{2ab}$ c) $2ab$ d) ab

121. If x_1,x_2,x_3,x_4 are the roots of the equation $x^4-x^3\sin 2\beta-x\cos\beta-\sin\beta=0$, then $\tan^{-1}+\tan^{-1}x_2+\tan^{-1}x_3+\tan^{-1}x_4$ is equal to

a) β b) $\frac{\pi}{2}-\beta$ c) $\pi-\beta$ d) $-\beta$

122. If $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, then the value of

$\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\sin 2x}{5+3\cos 2x}\right)$ is

a) $\frac{x}{2}$ b) $2x$ c) $3x$ d) x

123. $\frac{\alpha^3}{2}\operatorname{cosec}^2\left(\frac{1}{2}\tan^{-1}\ \frac{\alpha}{\beta}\right)+\frac{\beta^3}{2}\sec^2\left(\frac{1}{2}\tan^{-1}\ \left(\frac{\beta}{\alpha}\right)\right)$ is

a) $(\alpha-\beta)(\alpha^2+\beta^2)$ b) $(\alpha+\beta)(\alpha^2-\beta^2)$ c) $(\alpha+\beta)(\alpha^2+\beta^2)$ d) None of these

124. If $-1\le x\le 0$, then $\cos^{-1}(2x^2-1)$ equals

a) $2\cos^{-1}x$ b) $\pi-2\cos^{-1}x$ c) $2\pi-2\cos^{-1}x$ d) $-2\cos^{-1}x$

125. If $\cos^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1} x$, then x is equal to

a) 0 b) 1 c) -1 d) None of these

126. If $\sec^{-1} x = \text{cosec}^{-1} y$, then $\cos^{-1}\frac{1}{x} + \cos^{-1}\frac{1}{y} =$

a) π b) $\frac{\pi}{4}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{2}$

127. $\sec^2(\tan^{-1} 2) + \text{cosec}^2 (\cot^{-1} 3)$ is equal to

a) 1 b) 5 c) 10 d) 15

128. If $-1 \le x \le -\frac{1}{2}$, then $\sin^{-1}(3x - 4x^3)$ equals

a) $3\sin^{-1} x$ b) $\pi - 3\sin^{-1} x$ c) $-\pi - 3\sin^{-1} x$ d) None of these

129. $\tan\frac{2\pi}{5} - \tan\frac{\pi}{15} - \sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15}$ is equal to

a) $-\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) 1 d) $\sqrt{3}$

130. The value of $\tan\left\{\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right\}$ is

a) $\frac{2}{3\sqrt{5}}$ b) $\frac{2}{3}$ c) $\frac{1}{\sqrt{5}}$ d) $\frac{4}{\sqrt{5}}$

131. The value of

$\sin\left(\sin^{-1}\frac{1}{3} + \sec^{-1} 3\right) + \cos\left(\tan^{-1}\frac{1}{2} + \tan^{-1} 2\right)$ is

a) 1 b) 2 c) 3 d) 4

132. If $-\frac{1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}$, then $\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)$ equals

a) $3\tan^{-1} x$ b) $-\pi + 3\tan^{-1} x$ c) $\pi + 3\tan^{-1} x$ d) None of these

133. $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) =$

a) $-\frac{1}{\sqrt{10}}$ b) $\frac{1}{\sqrt{10}}$ c) $-\frac{1}{10}$ d) $\frac{1}{10}$

134. The solution of $\tan^{-1} 2x + \tan^{-1} 3x = \frac{\pi}{4}$ is

a) $\frac{1}{6}$ b) -1 c) $\left(\frac{1}{6}, -1\right)$ d) None of these

135. $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3}$ is equal to

a) $\frac{\pi}{3}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) 0

136. The equation $2\cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$ has

a) No solution b) Only one solution c) Two solutions d) Three solutions

137. The value of $\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\cos\frac{5\pi}{3}\right)$ is

a) $\frac{10\pi}{3}$ b) 0 c) $\frac{\pi}{2}$ d) $\frac{5\pi}{3}$

138. The value of $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right)$ is

a) 45° b) 90° c) 15° d) 30°

139. If $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$, then x equals

a) $1, -1$ b) $1, 0$ c) $0, \frac{1}{2}$ d) None of these

140. $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1} x\right), x \ne 0$ is equal to

a) x b) $2x$ c) $\frac{2}{x}$ d) None of these

141. $5\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 7\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\tan^{-1}\left(\frac{2x}{1-x^2}\right) - \tan^{-1} x = 5\pi$, then x is equal to

a) 3 b) $-\sqrt{3}$ c) $\sqrt{2}$ d) $\sqrt{3}$

142. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$ and $f(1)=2$, $f(p+q)=f(p).f(q), \forall\, p,q\in R$, then $x^{f(1)}+y^{f(2)}+z^{f(3)}-\frac{(x+y+z)}{x^{f(1)}+y^{f(2)}+z^{f(3)}}$ is equal to

a) 0 b) 1 c) 2 d) 3

143. $\cot^{-1}(\sqrt{\cos\alpha})-\tan^{-1}(\sqrt{\cos\alpha})=x$, then $\sin x$ is equal to

a) $\tan^{-2}\left(\frac{\alpha}{2}\right)$ b) $\cot^{2}\left(\frac{\alpha}{2}\right)$ c) $\tan\alpha$ d) $\cot\left(\frac{\alpha}{2}\right)$

144. The value of $\cot^{-1}9+\operatorname{cosec}^{-1}\frac{\sqrt{41}}{4}$ is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) π

145. $\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$ is equal to

a) $\tan^{-1}\left(\frac{n^2+n}{n^2+n+2}\right)$ b) $\tan^{-1}\left(\frac{n^2-n}{n^2-n+2}\right)$ c) $\tan^{-1}\left(\frac{n^2+n+2}{n^2+n}\right)$ d) None of these

146. If $\cos^{-1}\frac{3}{5}-\sin^{-1}\frac{4}{5}=\cos^{-1}x$, then x is equal to

a) 0 b) 1 c) -1 d) None of these

147. If $\cot(\cos^{-1}x)=\sec\left(\tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right)$, then x is equal to

a) $\frac{b}{\sqrt{2b^2-a^2}}$ b) $\frac{a}{\sqrt{2b^2-a^2}}$ c) $\frac{\sqrt{2b^2-a^2}}{a}$ d) $\frac{\sqrt{2b^2-a^2}}{b}$

148. The equation $\sin^{-1}x-\cos^{-1}x=\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ has

a) No solution b) Unique solution
c) Infinite number of solutions d) None of the above

149. If $\theta=\sin^{-1}x+\cos^{-1}x-\tan^{-1}x\geq 0$, then the smallest interval in which θ lies, is given by

a) $\frac{\pi}{2}\leq\theta\leq\frac{3\pi}{4}$ b) $-\frac{\pi}{4}\leq\theta\leq 0$ c) $0\leq\theta\leq\frac{\pi}{4}$ d) $\frac{\pi}{4}\leq\theta\leq\frac{\pi}{2}$

150. Solution of the equation $\cot^{-1}x+\sin^{-1}\frac{1}{\sqrt{5}}=\frac{\pi}{4}$ is

a) $x=3$ b) $x=\frac{1}{\sqrt{5}}$ c) $x=0$ d) None of these

151. $\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$ is equal to

a) $-\frac{1}{\sqrt{10}}$ b) $\frac{1}{\sqrt{10}}$ c) $-\frac{1}{10}$ d) $\frac{1}{10}$

152. If $\sin^{-1}\left(\frac{3}{x}\right)+\sin^{-1}\left(\frac{4}{x}\right)=\frac{\pi}{2}$, then x is equal to

a) 3 b) 5 c) 7 d) 11

153. If $[\cot^{-1}x]+[\cos^{-1}x]=0$, where x is a non-negative real number and [.] denotes the greatest integer function, then complete set of values of x is

a) $(\cos 1, 1]$ b) $(\cot 1, 1)$ c) $(\cos 1, \cot 1)$ d) None of these

154. If $3\sin^{-1}\frac{2x}{1+x^2}-4\cos^{-1}\frac{1+x}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$, then value of x is

a) $\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) 1 d) None of these

155. Sum of infinite terms of the series $\cot^{-1}\left(1^2+\frac{3}{4}\right)+\cot^{-1}\left(2^2+\frac{3}{4}\right)+\cot^{-1}\left(3^2+\frac{3}{4}\right)+\ldots$ is

a) $\frac{\pi}{4}$ b) $\tan^{-1}(2)$ c) $\tan^{-1}3$ d) None of these

156. $2\tan^{-1}\left(\frac{1}{3}\right)+\tan^{-1}\left(\frac{1}{7}\right)$ is equal to

a) $\left(\frac{49}{29}\right)$ b) $\frac{\pi}{2}$ c) $-\left(\frac{49}{29}\right)$ d) $\frac{\pi}{4}$

157. The number of triplets (x, y, z)satisfying $\sin^{-1} x + \cos^{-1} y + \sin^{-1} z = 2\pi$, is

a) 0 b) 2 c) 1 d) Infinite

158. The value of $\sin(\cot^{-1} x)$ is

a) $\sqrt{1+x^2}$ b) x c) $(1+x^2)^{-3/2}$ d) $(1+x^2)^{-1/2}$

159. If $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2}$ and $\tan^{-1} x - \tan^{-1} y = 0$, then $x^2 + xy + y^2$ is equal to

a) 0 b) $\frac{1}{\sqrt{2}}$ c) $\frac{3}{2}$ d) $\frac{1}{8}$

160. If $-1 < x < 1$, then $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ equals

a) $2\tan^{-1} x$ b) $-\pi + 2\tan^{-1} x$ c) $\pi + 2\tan^{-1} x$ d) None of these

161. If $\theta = \tan^{-1} a, \phi = \tan^{-1} b$ and $ab = -1$,then $(\theta - \phi)$ is equal to

a) 0 b) $\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) None of these

162. If the $(\cos^{-1} x) = \sin\left(\cot^{-1} \frac{1}{2}\right)$, then x is equal to

a) $\pm\frac{5}{3}$ b) $\pm\frac{\sqrt{5}}{3}$ c) $\pm\frac{5}{\sqrt{3}}$ d) None of these

163. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi$, then $x^4 + y^4 + z^4 + 4x^2y^2z^2 = k(x^2y^2 + y^2z^2 + z^2x^2)$ Where k is equal to

a) 1 b) 2 c) 4 d) none of these

164. $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) - \tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is equal to

a) 0 b) 1 c) $\tan^{-1} x$ d) $\tan^{-1} 2x$

165. If $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$, then the value of q is

a) 1 b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$

166. If α, β are the roots of the equation $6x^2 - 5x + 1 = 0$, then the value of $\tan^{-1}\alpha + \tan^{-1}\beta$ is

a) 0 b) $\pi/4$ c) 1 d) $\pi/2$

167. If $\alpha = \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{1}{3}$ and $\beta = \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{1}{3}$, then

a) $\alpha < \beta$ b) $\alpha = \beta$ c) $\alpha > \beta$ d) None of these

168. Solution set of $[\sin^{-1} x] > [\cos^{-1} x]$, white [.]denote the greatest integer function , is

a) $\left[\frac{1}{\sqrt{2}}, 1\right]$ b) $(\cos 1, \sin 1)$ c) $[\sin 1, 1]$ d) None of these

169. The value of $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$, is

a) 0 b) 1 c) π d) $-\pi$

170. The greatest and the least values of $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$ are respectively

a) $-\frac{\pi}{2}, \frac{\pi}{2}$ b) $-\frac{\pi^3}{8}, \frac{\pi^3}{8}$ c) $\frac{\pi^3}{32}, \frac{7\pi^3}{8}$ d) None of these

171. If $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{x}{3}\right) = 0$, then x is equal to

a) $\frac{7}{3}$ b) 3 c) $\frac{11}{3}$ d) $\frac{13}{3}$

172. Cos $[\tan^{-1}\{\sin(\cot^{-1} x)\}]$ is equal to

a) $\sqrt{\frac{x^2+2}{x^2+3}}$ b) $\sqrt{\frac{x^2+2}{x^2+1}}$ c) $\sqrt{\frac{x^2+1}{x^2+2}}$ d) None of these

173. $\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$ is equal to

(where $x < y > 0$)

a) $-\frac{\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{3\pi}{4}$ d) None of these

174. If $\tan^{-1} 2$ and $\tan^{-1} 3$ are two angles of a triangle, then the third angle is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

175. If θ and ϕ are the roots of the equation $8x^2 + 22x + 5 = 0$, then
a) Both $\sin^{-1}\theta$ and $\sin^{-1}\phi$ are equal
b) Both $\sec^{-1}\theta$ and $\sec^{-1}\phi$ are equal
c) Both $\tan^{-1}\theta$ and $\tan^{-1}\phi$ are equal
d) None of the above

176. If x_1, x_2, x_3, x_4 are the roots of the equation $x^4 - x^3 \sin 2\beta - x\cos\beta - \sin\beta = 0$, then $\tan^{-1} + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$ is equal to
a) β b) $\frac{\pi}{2} - \beta$ c) $\pi - \beta$ d) $-\beta$

177. The value of $\cos(2\cos^{-1}x + \sin^{-1}x)$ at $x = \frac{1}{5}$ is
a) 1 b) 3 c) 0 d) $-\frac{2\sqrt{6}}{5}$

178. Solution of the equation $\cot^{-1}x + \sin^{-1}\frac{1}{\sqrt{5}} = \frac{\pi}{4}$ is
a) $x = 3$ b) $x = \frac{1}{\sqrt{5}}$ c) $x = 0$ d) None of these

179. Let $\cos(2\tan^{-1}x) = \frac{1}{2}$, then the value of x is
a) $\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) $1 - \sqrt{3}$ d) $1 - \frac{1}{\sqrt{3}}$

180. If $\sin^{-1}\frac{1}{3} + \sin^{-1}\frac{2}{3} = \sin^{-1}x$, then the value of x is
a) 0 b) $\frac{(\sqrt{5} - 4\sqrt{2})}{9}$ c) $\frac{(\sqrt{5} + 4\sqrt{2})}{9}$ d) $\frac{\pi}{2}$

181. The solution set of the equation $\tan^{-1}x - \cot^{-1}x = \cos^{-1}(2 - x)$is
a) [0,1] b) [−1,1] c) [1,3] d) None of these

182. $\cos^{-1}\left\{\frac{1}{2}x^2 + \sqrt{1 - x^2}\sqrt{1 - \frac{x^2}{4}}\right\} = \cos^{-1}\frac{x}{2} - \cos^{-1}x$ holds for
a) $|x| \le 1$ b) $x \in R$ c) $0 \le x \le 1$ d) $-1 \le x \le 0$

183. The solutions of the equation $\tan^{-1}(x + 1) + \tan^{-1}(x - 1) = \tan^{-1}\frac{8}{31}$ are
a) $-\frac{1}{4}, 8$ b) $\frac{1}{4}, -8$ c) $-4, \frac{1}{8}$ d) $4, -\frac{1}{8}$

184. If $3\sin^{-1}\frac{2x}{1+x^2} - 4\cos^{-1}\frac{1+x}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$, then value of x is
a) $\sqrt{3}$ b) $\frac{1}{\sqrt{3}}$ c) 1 d) None of these

185. If $x^2 + y^2 + z^2 = r^2$, then
$\tan^{-1}\left(\frac{xy}{zr}\right) + \tan^{-1}\left(\frac{yz}{xr}\right) + \tan^{-1}\left(\frac{xz}{yr}\right)$ is equal to
a) π b) $\frac{\pi}{2}$ c) 0 d) None of these

186. The greatest and the least values of $(\sin^{-1}x)^3 + (\cos^{-1}x)^3$ are respectively
a) $-\frac{\pi}{2}, \frac{\pi}{2}$ b) $-\frac{\pi^3}{8}, \frac{\pi^3}{8}$ c) $\frac{\pi^3}{32}, \frac{7\pi^3}{8}$ d) None of these

187. For the principle value branch of the graph of the function $y = \sin^{-1}x, -1 \le x \le 1$, which among the following is a true statement?
a) Graph is symmetric about the x-axis
b) Graph is symmetric about the y-axis
c) Graph is not continuous
d) The line $x = 1$ is a tangent

188. If $-1 \le x \le -\frac{1}{\sqrt{2}}$, then $\sin^{-1}\left(2x\sqrt{1 - x^2}\right)$ equals
a) $2\sin^{-1}x$ b) $\pi - 2\sin^{-1}x$ c) $-\pi - 2\sin^{-1}x$ d) None of these

189. If a, b, c be positive real number and the value of

$$\theta=\tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}}+\tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}}+\tan^{-1}\sqrt{\frac{c(c+b+c)}{ab}}$$

Then $\tan\theta$ is equal to

a) 0 b) 1 c) $\frac{a+b+c}{abc}$ d) None of these

190. If $\theta\in[4\pi,5\pi]$, then $\cos^{-1}(\cos\theta)$ equals

a) $-4\pi+\theta$ b) $5\pi-\theta$ c) $4\pi-\theta$ d) $\theta-5\pi$

191. The trigonometric equation $\sin^{-1}x=2\sin^{-1}a$, has a solution for

a) $\frac{1}{2}<|a|<\frac{1}{\sqrt{2}}$ b) All real values of a c) $|a|\le\frac{1}{2}$ d) $|a|\ge\frac{1}{\sqrt{2}}$

192. The number of solutions of the equation $\sin^{-1}x+\sin^{-1}2x=\frac{\pi}{3}$, is

a) 0 b) 1 c) 2 d) Infinite

193. If $2\sin^{-1}x=\sin^{-1}(2x\sqrt{1-x^2})$, then x is equal to

a) $[-1,1]$ b) $\left[-\frac{1}{\sqrt{2}},1\right]$ c) $\left[-\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right]$ d) None of these

194. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$, then

a) $x^2+y^2=z^2$ b) $x^2+y^2+z^2=0$
c) $x^2+y^2+z^2=1-2xyz$ d) None of the above

195. The value of $\cot\left(\operatorname{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$ is

a) $\frac{5}{17}$ b) $\frac{6}{17}$ c) $\frac{3}{17}$ d) $\frac{4}{17}$

196. If $\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\pi$, then $x^4+y^4+z^4+4x^2y^2z^2=k(x^2y^2+y^2z^2+z^2x^2)$ Where k is equal to

a) 1 b) 2 c) 4 d) none of these

197. If $\tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$, then the value of $x+y+z$ is

a) $-xyz$ b) xyz c) $\frac{1}{xyz}$ d) 0

198. The value of $\cos^{-1}(\cos 12)-\sin^{-1}(\sin 14)$ is

a) 2 b) $8\pi-26$ c) $4\pi+2$ d) None of these

199. If $\frac{1}{2}\le x\le 1$, then $\sin^{-1}(3x-4x^3)$ equals

a) $3\sin^{-1}x$ b) $\pi-3\sin^{-1}x$ c) $-\pi-3\sin^{-1}x$ d) None of these

200. The value of $\sin^{-1}\{\cos(4095°)\}$ is equal to

a) $-\frac{\pi}{3}$ b) $\frac{\pi}{6}$ c) $-\frac{\pi}{4}$ d) $\frac{\pi}{4}$

201. $\tan^{-1}\left(\frac{1}{4}\right)+\tan^{-1}\left(\frac{2}{9}\right)=$

a) $\frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$ b) $\frac{1}{2}\sin^{-1}\left(\frac{3}{5}\right)$ c) $\frac{1}{2}\tan^{-1}\left(\frac{3}{5}\right)$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

202. If $\sin^{-1}\alpha+\sin^{-1}\beta+\sin^{-1}\gamma=\frac{3\pi}{2}$, then $\alpha\beta+\alpha\gamma+\beta\gamma$ is equal to

a) 1 b) 0 c) 3 d) -3

203. If $A=\tan^{-1}\left(\frac{x\sqrt{3}}{2k-x}\right)$ and $B=\tan^{-1}\left(\frac{2x-k}{k\sqrt{3}}\right)$,then the value of $A-B$ is

a) 10° b) 45° c) 60° d) 30°

204. If in a ΔABC, $\angle A=\tan^{-1}2$ and $\angle B=\tan^{-1}3$, then angle C is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) None of these

205. If $(\tan^{-1}x)^2+(\cot^{-1}x)^2=\frac{5\pi^2}{8}$, then x equals

a) -1 b) 1 c) 0 d) None of these

206. $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}$ is equal to

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{4}$

207. If $\sin^{-1}x-\cos^{-1}x=\frac{\pi}{6}$, then x is

a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $\frac{-1}{2}$ d) None of these

208. If the mapping $f(x)=ax+b, a>0$ maps $[-1,1]$ onto $[0, 2]$ then $\cot[\cot^{-1}7+\cot^{-1}8+\cot^{-1}18]$ is equal to

a) $f(-1)$ b) $f(0)$ c) $f(1)$ d) $f(2)$

209. The value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$ is

a) $\frac{3\pi}{5}$ b) $\frac{7\pi}{5}$ c) $\frac{\pi}{10}$ d) $-\frac{\pi}{10}$

210. For the equation $\cos^{-1}x+\cos^{-1}2x+\pi=0$, then the number of real solutions is

a) 1 b) 2 c) 0 d) ∞

211. The value of $\tan\left\{\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right\}$, is

a) $\frac{3+\sqrt{5}}{2}$ b) $3+\sqrt{5}$ c) $\frac{1}{2}(3-\sqrt{5})$ d) None of these

212. The value of $\sin\left[2\cos^{-1}\frac{\sqrt{5}}{3}\right]$ is

a) $\frac{\sqrt{5}}{3}$ b) $\frac{2\sqrt{5}}{3}$ c) $\frac{4\sqrt{5}}{9}$ d) $\frac{2\sqrt{5}}{9}$

213. If $x>-\frac{1}{\sqrt{3}}$, then $\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ equals

a) $3\tan^{-1}x$ b) $-\pi+3\tan^{-1}x$ c) $\pi+3\tan^{-1}x$ d) None of these

214. $\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$ is equal to

a) $\frac{\pi}{6}$ b) $\frac{\pi}{3}$ c) $\frac{2\pi}{3}$ d) $\frac{\pi}{4}$

215. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then the value of x is

a) -1 b) 2/5 c) 1/3 d) 1/5

216. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$, then $xy+yz+zx$ is equal to

a) 0 b) 1 c) 3 d) -3

217. If $0\le x<\infty$, then $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ equals

a) $2\tan^{-1}x$ b) $-2\tan^{-1}x$ c) $\pi-2\tan^{-1}x$ d) $\pi+2\tan^{-1}x$

218. The value of $\cos[2\tan^{-1}(-7)]$ is

a) $\frac{49}{50}$ b) $-\frac{49}{50}$ c) $\frac{24}{25}$ d) $-\frac{24}{25}$

219. The value of $\sin\left(4\tan^{-1}\frac{1}{3}\right)-\cos\left(2\tan^{-1}\frac{1}{7}\right)$ is

a) $\frac{3}{7}$ b) $\frac{7}{8}$ c) $\frac{8}{21}$ d) None of these

220. If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=3\pi$, then $xy+yz+zx$ is equal to

a) 0 b) 1 c) 3 d) -3

221. If $a_1, a_2, a_3, \dots, a_n$ are in AP with common ratio d, then

$\tan\left[\tan^{-1}\frac{d}{1+a\_1a_2}+\tan^{-1}\frac{d}{1+a_2a_3}+\dots+\tan^{-1}\frac{4}{1+a_{n-1}a_n}\right]$ is equal to

a) $\frac{(n-1)d}{a_1+a_n}$ b) $\frac{(n-1)d}{1+a_1a_n}$ c) $\frac{nd}{1+a_1a_n}$ d) $\frac{a_n-a_1}{a_n+a_1}$

222. $\sin\left(2\sin^{-1}\sqrt{\frac{63}{65}}\right)$ is equal to

a) $\frac{2\sqrt{126}}{65}$ b) $\frac{4\sqrt{65}}{65}$ c) $\frac{8\sqrt{63}}{65}$ d) $\frac{\sqrt{63}}{65}$

223. If $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$, then x is

a) $\frac{1}{2}$ b) $\frac{\sqrt{3}}{2}$ c) $-\frac{1}{2}$ d) None of these

224. If $\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 4\tan^{-1} x$, then

a) $x \in -(-\infty, -1)$ b) $x \in (1, \infty)$ c) $x \in [0, 1]$ d) $x \in [-1, 0]$

225. $\tan^{-1}\frac{c_1x-y}{c_1y+x} + \tan^{-1}\frac{c_2-c_1}{1+c_2c_1} + \tan^{-1}\frac{c_3-c_2}{1+c_3c_2} + \ldots + \tan^{-1}\frac{1}{c_n}$ is equal to

a) $\tan^{-1}\frac{y}{x}$ b) $\tan^{-1} yx$ c) $\tan^{-1}\frac{x}{y}$ d) $\tan^{-1}(x-y)$

226. If $\tan^{-1} a + \tan^{-1} b = \sin^{-1} 1 - \tan^{-1} c$, then

a) $a + b + c = abc$

b) $ab + bc + ca = abc$

c) $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} - \frac{1}{abc} = 0$

d) $ab + bc + ca = a + b + c$

227. The value of $\cos[\tan^{-1}\{\sin(\cot^{-1} x)\}]$ is

a) $\sqrt{\frac{x^2+1}{x^2-1}}$ b) $\sqrt{\frac{1-x^2}{x^2+2}}$ c) $\sqrt{\frac{1-x^2}{1+x^2}}$ d) $\sqrt{\frac{x^2+1}{x^2+2}}$

228. If $[\cot^{-1} x] + [\cos^{-1} x] = 0$, where x is a non-negative real number and [.] denotes the greatest integer function, then complete set of values of x is

a) $(\cos 1, 1]$ b) $(\cot 1, 1)$ c) $(\cos 1, \cot 1)$ d) None of these

229. If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$, then $xy + yz + zx$ is equal to

a) 1 b) 0 c) -3 d) 3

230. A solution of the equation $\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$, is

a) $x = 1$ b) $x = -1$ c) $x = 0$ d) $x = \pi$

231. $\tan\left(\frac{\pi}{4} + \frac{1}{2}\cos^{-1} x\right) + \tan\left(\frac{\pi}{4} - \frac{1}{2}\cos^{-1} x\right), x \neq 0$ is equal to

a) x b) $2x$ c) $\frac{2}{x}$ d) None of these

232. The equation $2\cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$ has

a) No solution b) Only one solution c) Two solutions d) Three solutions

233. If $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$, then the value of $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101}+y^{101}+z^{101}}$, is

a) 0 b) 1 c) 2 d) 3

234. If $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1} x\right) = 1$, then x is equal to

a) 1 b) 0 c) 4/5 d) 1/5

235. If the mapping $f(x) = ax + b, a > 0$ maps $[-1,1]$ onto $[0, 2]$ then $\cot[\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18]$ is equal to

a) $f(-1)$ b) $f(0)$ c) $f(1)$ d) $f(2)$

236. If $\sin^{-1}\frac{2a}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\frac{2x}{1-x^2}$, then value of x is

a) a b) b c) $\frac{a+b}{1-ab}$ d) $\frac{a-b}{1+ab}$

237. The sum of the two angles $\cot^{-1} 3$ and $\text{cosec}^{-1}\sqrt{5}$, is

a) $\frac{\pi}{2}$ b) $\frac{\pi}{3}$ c) $\frac{\pi}{4}$ d) $\frac{\pi}{6}$

238. $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$ is equal to

a) $\frac{2a}{1+a^2}$ b) $\frac{1-a^2}{1+a^2}$ c) $\frac{2a}{1-a^2}$ d) None of these

239. If $\tan^{-1} x + \tan^{-1} y = \frac{\pi}{4}$, then

a) $x + y + xy = 1$ b) $x + y - xy = 1$
c) $x + y + xy + 1 = 0$ d) $x + y - xy + 1 = 0$

240. If $0 \leq x \leq 1$, then $\cos^{-1}(2x^2 - 1)$ equals

a) $2\cos^{-1} x$ b) $\pi - 2\cos^{-1} x$ c) $2\pi - 2\cos^{-1} x$ d) None of these

241. If a, b, c be positive real number and the value of

$$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(c+b+c)}{ab}}$$

Then $\tan\theta$ is equal to

a) 0 b) 1 c) $\frac{a+b+c}{abc}$ d) None of these

242. If $\sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$, then $\cos^{-1} x + \cos^{-1} y$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{\pi}{4}$ c) π d) $\frac{3\pi}{4}$

243. $\tan^{-1}\frac{c_1 x - y}{c_1 y + x} + \tan^{-1}\frac{c_2 - c_1}{1 + c_2 c_1} + \tan^{-1}\frac{c_3 - c_2}{1 + c_3 c_2} + \ldots + \tan^{-1}\frac{1}{c_n}$ is equal to

a) $\tan^{-1}\frac{y}{x}$ b) $\tan^{-1} yx$ c) $\tan^{-1}\frac{x}{y}$ d) $\tan^{-1}(x - y)$

244. The value of $\cos\{\tan^{-1}(\tan 2)\}$, is

a) $\frac{1}{\sqrt{5}}$ b) $-\frac{1}{\sqrt{5}}$ c) $\cos 2$ d) $-\cos 2$

245. If $\tan^{-1}\frac{x-1}{x+2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$, then x is equal to

a) $\frac{1}{\sqrt{2}}$ b) $-\frac{1}{\sqrt{2}}$ c) $\pm\sqrt{\frac{5}{2}}$ d) $\pm\frac{1}{2}$

246. The sum of series

$$\tan^{-1}\frac{1}{1+1+1^2} + \tan^{-1}\frac{1}{1+2+2^2} + \tan^{-1}\frac{1}{1+3+3^2} + \cdots$$

∞ is equal to

a) $\frac{\pi}{4}$ b) $\frac{\pi}{2}$ c) $\frac{\pi}{3}$ d) $\frac{\pi}{6}$

247. The value of $'a'$ for which $ax^2 + \sin^{-1}(x^2 - 2x + 2) + \cos^{-1}(x^2 - 2x + 2) = 0$ has a real solution, is

a) $-\frac{2}{\pi}$ b) $\frac{2}{\pi}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{2}$

248. If $\tan^{-1} x - \tan^{-1} y = \tan^{-1} A$, then A is equal to

a) $x - y$ b) $x + y$ c) $\frac{x-y}{1+xy}$ d) $\frac{x+y}{1-xy}$

249. If $\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$, then x is equal to

a) $\sqrt{ab}$ b) $\sqrt{2ab}$ c) $2ab$ d) ab

250. $\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2}$ is equal to

a) $\frac{\pi}{4}$ b) $\frac{\pi}{6}$ c) $\frac{\pi}{3}$ d) $\frac{2\pi}{3}$

251. $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}$ is equal to

a) π b) $\pi/2$ c) $\pi/3$ d) $\pi/4$

252. If $2\sin^{-1} x = \sin^{-1}(2x\sqrt{1 - x^2})$, then x is equal to

a) $[-1,1]$ b) $\left[-\frac{1}{\sqrt{2}}, 1\right]$ c) $\left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$ d) None of these

253. The value of $\cot\left(\text{cosec}^{-1}\frac{5}{3}+\tan^{-1}\frac{2}{3}\right)$, is

a) $\frac{4}{17}$ b) $\frac{5}{17}$ c) $\frac{6}{17}$ d) $\frac{3}{17}$

254. Two angles of a triangle are $\cot^{-1}2$ and $\cot^{-1}3$. Then , the third angle is

a) $\frac{\pi}{4}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{6}$ d) $\frac{\pi}{3}$

255. If $e^{[\sin^2\alpha+\sin^4\alpha+\sin^6\alpha+\dots\infty]\log_e 2}$ is a root of equation $x^2-9x+8=0$,where $0<\alpha<\frac{\pi}{2}$, then the principle value of $\sin^{-1}\sin\left(\frac{2\pi}{3}\right)$ is

a) α b) 2α c) $-\alpha$ d) -2α

256. If $\frac{1}{2}\le x\le 1$, then $\cos^{-1}(4x^3-3x)$ equals

a) $3\cos^{-1}x$ b) $2\pi-3\cos^{-1}x$ c) $-2\pi-3\cos^{-1}x$ d) None of these

257. If $\sin^{-1}\left(2x\sqrt{1-x^2}\right)-2\sin^{-1}x=0$, then x belongs to the interval

a) $[-1,1]$ b) $[-1/\sqrt{2},1/\sqrt{2}]$ c) $[-1,-1/\sqrt{2}]$ d) $[1/\sqrt{2},1]$

258. Solution set of $[\sin^{-1}x]>[\cos^{-1}x]$, white [.]denote the greatest integer function , is

a) $\left[\frac{1}{\sqrt{2}},1\right]$ b) $(\cos 1,\sin 1)$ c) $[\sin 1,1]$ d) None of these

259. If $[\sin^{-1}\cos^{-1}\sin^{-1}x]=1$, where [.] denotes the greatest integer function, then x belongs to the interval

a) [tan sin cos 1, tan sin cos sin 1] b) (tan sin cos 1, tan sin cos sin 1)
c) $[-1,1]$ d) [sin cos tan 1, sin cos sin tan 1]

260. The solution of $\tan^{-1}x+2\cot^{-1}x=\frac{2\pi}{3}$ is

a) $-\frac{1}{\sqrt{3}}$ b) $\frac{1}{\sqrt{3}}$ c) $-\sqrt{3}$ d) $\sqrt{3}$

261. If $x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$, then the value of $\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\sin 2x}{5+3\cos 2x}\right)$ is

a) $\frac{x}{2}$ b) $2x$ c) $3x$ d) x

262. If $-\frac{1}{2}\le x\le\frac{1}{2}$, then $\sin^{-1}(3x-4x^3)$ equals

a) $3\sin^{-1}x$ b) $\pi-3\sin^{-1}x$ c) $-\pi-3\sin^{-1}x$ d) None of these

263. If $\tan\theta+\tan\left(\frac{\pi}{3}+\theta\right)+\tan\left(\frac{-\pi}{3}+\theta\right)=K\tan 3\theta$, then the value of K is

a) 1 b) 1/3 c) 3 d) none of these

264. If $-1\le x\le 0$, then $\cos^{-1}(2x^2-1)$ equals

a) $2\cos^{-1}xx$ b) $\pi-2\cos^{-1}x$ c) $2\pi-2\cos^{-1}x$ d) $-2\cos^{-1}x$

265. If $\alpha=\sin^{-1}\frac{\sqrt{3}}{2}+\sin^{-1}\frac{1}{3}$, $\beta=\cos^{-1}\frac{\sqrt{3}}{2}+\cos^{-1}\frac{1}{3}$, then

a) $\alpha>\beta$ b) $\alpha=\beta$ c) $\alpha<\beta$ d) $\alpha+\beta=2\pi$

266. If $x\in[-1,1]$, then $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ equals

a) $2\tan^{-1}x$ b) $\pi-2\tan^{-1}x$ c) $-\pi-2\tan^{-1}x$ d) None of these

267. $\sin\left[3\sin^{-1}\left(\frac{1}{5}\right)\right]$ is equal to

a) $\frac{71}{125}$ b) $\frac{74}{125}$ c) $\frac{3}{5}$ d) $\frac{1}{2}$

268. If $\sum_{i=1}^{20}\sin^{-1}x_i=10\pi$,then $\sum_{i=1}^{20}x_i$ is equal to

a) 20 b) 10 c) 0 d) None of these

269. The value of x for which $\sin[\cot^{-1}(1+x)]=\cos(\tan^{-1}x)$ is

a) $\frac{1}{2}$ b) 1 c) 0 d) $-\frac{1}{2}$

270. $\tan\left[\frac{\pi}{2}+\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]$ is equal to

a) $\frac{2a}{b}$ b) $\frac{2b}{a}$ c) $\frac{a}{b}$ d) $\frac{b}{a}$

271. $\tan^{-1}\frac{x}{y}-\tan^{-1}\frac{x-y}{x+y}$ is equal to (where $x<y>0$)

a) $-\frac{\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{3\pi}{4}$ d) None of these

272. The value of $'a'$ for which $ax^2+\sin^{-1}(x^2-2x+2)+\cos^{-1}(x^2-2x+2)=0$ has a real solution, is

a) $-\frac{2}{\pi}$ b) $\frac{2}{\pi}$ c) $-\frac{\pi}{2}$ d) $\frac{\pi}{2}$

273. $\cos^{-1}\left(\frac{-1}{2}\right)-2\sin^{-1}\left(\frac{1}{2}\right)+3\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right)-4\tan^{-1}(-1)$ equals

a) $\frac{19\pi}{12}$ b) $\frac{35\pi}{12}$ c) $\frac{47\pi}{12}$ d) $\frac{43\pi}{12}$

274. If $\theta=\sin^{-1}x+\cos^{-1}x-\tan^{-1}x, 1\le x<\infty$, then the smallest interval in which θ lies is

a) $\frac{\pi}{2}\le\theta\le\frac{3\pi}{4}$ b) $0\le\theta\le\frac{\pi}{4}$ c) $-\frac{\pi}{4}\le\theta\le 0$ d) $\frac{\pi}{4}\le\theta\le\frac{\pi}{2}$

275. If $4\sin^{-1}x+\cos^{-1}x=\pi$, then x is equal to

a) 0 b) 1/2 c) -1/2 d) 1

276. The value of $\sin^{-1}\left(\cos\frac{33\pi}{5}\right)$ is

a) $\frac{3\pi}{5}$ b) $\frac{7\pi}{5}$ c) $\frac{\pi}{10}$ d) $-\frac{\pi}{10}$

277. If $a_1, a_2, a_3, \dots, a_n$ are in AP with common ratio d, then $\tan\left[\tan^{-1}\frac{d}{1+a\_1a_2}+\tan^{-1}\frac{d}{1+a_2a_3}+\dots+\tan^{-1}\frac{4}{1+a_{n-1}a_n}\right]$ is equal to

a) $\frac{(n-1)d}{a_1+a_n}$ b) $\frac{(n-1)d}{1+a_1a_n}$ c) $\frac{nd}{1+a_1a_n}$ d) $\frac{a_n-a_1}{a_n+a_1}$

278. If $\tan^{-1}\left(\frac{a}{x}\right)+\tan^{-1}\left(\frac{b}{x}\right)=\frac{\pi}{2}$, then x is equal to

a) $\sqrt{ab}$ b) $\sqrt{2ab}$ c) $2ab$ d) ab

279. If $A=\tan^{-1}x, x\in R$, then the value of $\sin 2A$ is

a) $\frac{2x}{1-x^2}$ b) $\frac{2x}{\sqrt{1-x^2}}$ c) $\frac{2x}{1+x^2}$ d) $\frac{1-x^2}{1+x^2}$

280. The value of x, where $x>0$ and $\tan\left\{\sec^{-1}\left(\frac{1}{x}\right)\right\}=\sin(\tan^{-1}2)$ is

a) $\sqrt{5}$ b) $\frac{\sqrt{5}}{3}$ c) 1 d) $\frac{2}{3}$

281. If $a<\frac{1}{32}$, then the number of solutions of $(\sin^{-1}x)^3+(\cos^{-1}x)^3=a\pi^3$, is

a) 0 b) 1 c) 2 d) Infinite

282. If $\sqrt{3}+i=(a+ib)(c+id)$, then $\tan^{-1}\left(\frac{b}{a}\right)+\tan^{-1}\left(\frac{d}{c}\right)$ has the value

a) $\frac{\pi}{3}+2n\pi, n\in I$ b) $n\pi+\frac{\pi}{6}, n\in I$ c) $n\pi-\frac{\pi}{3}, n\in I$ d) $2n\pi-\frac{\pi}{3}, n\in I$

283. If $\tan^{-1}\frac{1-x}{1+x}=\frac{1}{2}\tan^{-1}x$, then the value of x is

a) $\frac{1}{2}$ b) $\frac{1}{\sqrt{3}}$ c) $\sqrt{3}$ d) 2

284. The sum of the infinite series $\cot^{-1}2+\cot^{-1}8+\cot^{-1}18+\cot^{-1}32+\dots$ is

a) π b) $\frac{\pi}{2}$ c) $\frac{\pi}{4}$ d) None of these

285. If $y=\cos^{-1}(\cos 10)$, then y is equal to

a) 10 b) $4\pi-10$ c) $2\pi+10$ d) $2\pi-10$

286. The principle value of $\sin^{-1}\tan\left(\frac{-5\pi}{4}\right)$ is

a) $\frac{\pi}{4}$ b) $-\frac{\pi}{4}$ c) $\frac{\pi}{2}$ d) $-\frac{\pi}{2}$

287. The value of $\sum_{r=0}^{\infty} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)$ is equal to

a) $\frac{\pi}{2}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{4}$ d) None of these

288. If $-1 \le x \le -\frac{1}{2}$, then $\cos^{-1}(4x^3 - 3x)$ equals

a) $3\cos^{-1}x$ b) $2\pi - 3\cos^{-1}x$ c) $-2\pi + 3\cos^{-1}x$ d) None of these

289. If $\tan^{-1}x - \tan^{-1}y = \tan^{-1}A$, then A is equal to

a) $x - y$ b) $x + y$ c) $\frac{x-y}{1+xy}$ d) $\frac{x+y}{1-xy}$

290. If $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$, then the value of x is

a) $\frac{3\pi}{4}$ b) $\frac{\pi}{4}$ c) $\frac{\pi}{3}$ d) None of these

291. The number of real solution of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is

a) 0 b) 1 c) 2 d) ∞

292. $\cos\left\{\cos^{-1}\left(-\frac{1}{7}\right) + \sin^{-1}\left(-\frac{1}{7}\right)\right\} =$

a) $-\frac{1}{3}$ b) 0 c) $\frac{1}{3}$ d) $\frac{4}{9}$

293. The number of triplets (x, y, z) satisfying $\sin^{-1}x + \cos^{-1}y + \sin^{-1}z = 2\pi$, is

a) 0 b) 2 c) 1 d) Infinite

294. If $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$, then x equals

a) $0, -\frac{1}{2}$ b) $0, \frac{1}{2}$ c) 0 d) None of these

295. $\cos^{-1}\left(\frac{15}{17}\right) + 2\tan^{-1}\left(\frac{1}{5}\right) =$

a) $\frac{\pi}{2}$ b) $\cos^{-1}\left(\frac{171}{221}\right)$ c) $\frac{\pi}{4}$ d) None of these

296. If $x < 0$, then $\tan^{-1}\left(\frac{1}{x}\right)$ equals

a) $\cot^{-1}x$ b) $-\cot^{-1}x$ c) $-\pi + \cot^{-1}x$ d) $-\pi - \cot^{-1}x$

297. If $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p} + \cos^{-1}\sqrt{1-q} = \frac{3\pi}{4}$, then the value of q is

a) 1 b) $\frac{1}{\sqrt{2}}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$

298. $\tan^{-1}\frac{m}{n} - \tan^{-1}\frac{m-n}{m+n}$ is equal to

a) $\tan^{-1}\frac{n}{m}$ b) $\tan^{-1}\frac{m+n}{m-n}$ c) $\frac{\pi}{4}$ d) $\tan^{-1}\left(\frac{1}{2}\right)$

299. The value of $\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$ is

a) $\frac{\sqrt{3}}{2}$ b) $-\frac{\sqrt{3}}{2}$ c) $\frac{1}{2}$ d) $-\frac{1}{2}$

300. If $\cos^{-1}x > \sin^{-1}x$, then

a) $x < 0$ b) $-1 < x < 0$ c) $0 \le x < \frac{1}{\sqrt{2}}$ d) $-1 \le x < \frac{1}{\sqrt{2}}$

301. The value of $\cot^{-1}\frac{xy+1}{x-y} + \cot^{-1}\frac{yz+1}{y-z} + \cot^{-1}\frac{zx+1}{z-x}$ is

a) 0 b) 1

c) $\cot^{-1}x + \cot^{-1}y + \cot^{-1}z$ d) None of the above

302. The value of $\tan\left\{\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right\}$ is

a) $\frac{2}{3\sqrt{5}}$ b) $\frac{2}{3}$ c) $\frac{1}{\sqrt{5}}$ d) $\frac{4}{\sqrt{5}}$

303. $\sin^{-1} x + \sin^{-1}\frac{1}{x} + \cos^{-1} x + \cos^{-1}\frac{1}{x}$ is equal to

a) π b) $\frac{\pi}{2}$ c) $\frac{3\pi}{2}$ d) None of these

304. If $x = \sin(2\tan^{-1} 2)$ and $y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{3}\right)$, then

a) $x = y^2$ b) $y^2 = 1 - x$ c) $x^2 = \frac{y}{2}$ d) $y^2 = 1 + x$

305. The simplified expression of $\sin(\tan^{-1} x)$, for any real number x is given by

a) $\frac{1}{\sqrt{1+x^2}}$ b) $\frac{x}{\sqrt{1+x^2}}$ c) $-\frac{1}{\sqrt{1+x^2}}$ d) $-\frac{x}{\sqrt{1+x^2}}$

306. The number of real solutions of $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{2}$ is

a) 0 b) 1 c) 2 d) ∞

307. If $-\infty < x \leq 0$, then $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ equals

a) $2\tan^{-1} x$ b) $-2\tan^{-1} x$ c) $\pi - 2\tan^{-1} x$ d) $\pi + 2\tan^{-1} x$

308. If $-\frac{\pi}{2} < \sin^{-1} x < \frac{\pi}{2}$, then $\tan(\sin^{-1} x)$ is equal to

a) $\frac{x}{1-x^2}$ b) $\frac{x}{1+x^2}$ c) $\frac{x}{\sqrt{1-x^2}}$ d) $\frac{1}{\sqrt{1-x^2}}$

INVERSE TRIGONOMETRICE FUNCTIONS

: ANSWER KEY :

1)	a	2)	d	3)	b	4)	c	157)	c	158)	d	159)	c	160)	a
5)	c	6)	c	7)	c	8)	d	161)	c	162)	b	163)	b	164)	c
9)	a	10)	c	11)	b	12)	c	165)	d	166)	b	167)	a	168)	c
13)	a	14)	b	15)	d	16)	d	169)	c	170)	d	171)	d	172)	c
17)	b	18)	c	19)	a	20)	c	173)	b	174)	c	175)	c	176)	b
21)	a	22)	b	23)	c	24)	b	177)	d	178)	a	179)	b	180)	c
25)	a	26)	b	27)	d	28)	a	181)	c	182)	a	183)	b	184)	b
29)	d	30)	d	31)	c	32)	a	185)	b	186)	d	187)	d	188)	c
33)	c	34)	d	35)	a	36)	d	189)	a	190)	a	191)	c	192)	b
37)	a	38)	b	39)	d	40)	a	193)	c	194)	c	195)	b	196)	b
41)	d	42)	c	43)	a	44)	d	197)	b	198)	b	199)	b	200)	c
45)	a	46)	a	47)	a	48)	a	201)	d	202)	c	203)	d	204)	c
49)	a	50)	d	51)	a	52)	c	205)	a	206)	d	207)	b	208)	d
53)	b	54)	d	55)	b	56)	c	209)	d	210)	c	211)	c	212)	c
57)	b	58)	a	59)	c	60)	c	213)	c	214)	c	215)	d	216)	c
61)	d	62)	a	63)	c	64)	b	217)	a	218)	d	219)	d	220)	c
65)	d	66)	b	67)	d	68)	c	221)	b	222)	a	223)	b	224)	c
69)	c	70)	c	71)	a	72)	c	225)	c	226)	c	227)	d	228)	b
73)	c	74)	b	75)	a	76)	b	229)	d	230)	c	231)	c	232)	a
77)	c	78)	a	79)	c	80)	b	233)	a	234)	d	235)	d	236)	d
81)	d	82)	c	83)	b	84)	c	237)	c	238)	c	239)	a	240)	a
85)	c	86)	d	87)	b	88)	b	241)	a	242)	a	243)	c	244)	d
89)	b	90)	a	91)	d	92)	d	245)	c	246)	a	247)	c	248)	c
93)	c	94)	b	95)	b	96)	a	249)	a	250)	d	251)	d	252)	c
97)	c	98)	c	99)	a	100)	a	253)	c	254)	b	255)	a	256)	a
101)	a	102)	b	103)	b	104)	b	257)	b	258)	c	259)	a	260)	d
105)	c	106)	c	107)	b	108)	c	261)	d	262)	a	263)	c	264)	d
109)	a	110)	d	111)	c	112)	b	265)	c	266)	a	267)	a	268)	a
113)	c	114)	a	115)	a	116)	c	269)	d	270)	b	271)	b	272)	c
117)	d	118)	c	119)	c	120)	a	273)	d	274)	b	275)	b	276)	d
121)	b	122)	d	123)	c	124)	c	277)	b	278)	a	279)	c	280)	b
125)	b	126)	d	127)	d	128)	c	281)	a	282)	b	283)	b	284)	c
129)	d	130)	a	131)	a	132)	a	285)	b	286)	d	287)	a	288)	c
133)	b	134)	a	135)	c	136)	a	289)	c	290)	b	291)	c	292)	b
137)	c	138)	d	139)	c	140)	c	293)	c	294)	c	295)	d	296)	c
141)	d	142)	c	143)	a	144)	b	297)	d	298)	c	299)	c	300)	d
145)	a	146)	b	147)	a	148)	b	301)	a	302)	a	303)	a	304)	b
149)	d	150)	a	151)	b	152)	b	305)	b	306)	c	307)	b	308)	c
153)	b	154)	b	155)	b	156)	d								

INVERSE TRIGONOMETRICE FUNCTIONS

: HINTS AND SOLUTIONS :

1 **(a)**

We have, $1 \le \sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1}x \le \frac{\pi}{2}$

$\Rightarrow \sin 1 \le \cos^{-1}\sin^{-1}\tan^{-1}x \le 1$

$\Rightarrow \cos\sin 1 \ge \sin^{-1}\tan^{-1}x \ge \cos 1$

$\Rightarrow \sin\cos\sin 1 \ge \tan^{-1}x \ge \sin\cos 1$

$\Rightarrow \tan\sin\cos\sin 1 \ge x \ge \tan\sin\cos 1$

$\therefore x \in [\tan\sin\cos 1, \tan\sin\cos\sin 1]$

3 **(b)**

Given, $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\therefore \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \frac{2\cos x}{1-\cos^2 x} = 2\operatorname{cosec} x$

$\Rightarrow \frac{2\cos x}{\sin^2 x} = 2\operatorname{cosec} x$

$\Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$

4 **(c)**

We have, $\tan^{-1}\frac{x-1}{x+2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x+2}+\frac{x+1}{x+2}}{1-\left(\frac{x-1}{x+2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$

$\Rightarrow \left[\frac{2x(x+2)}{x^2+4+4x-x^2+1}\right] = \tan\frac{\pi}{4}$

$\Rightarrow \frac{2x(x+2)}{4x+5} = 1$

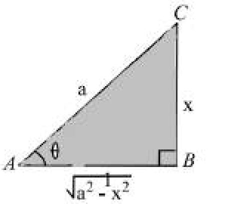
$\Rightarrow 2x^2+4x = 4x+5$

$\Rightarrow x = \pm\sqrt{\frac{5}{2}}$

5 **(c)**

Let $\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} = \theta$

$\Rightarrow \tan\theta = \frac{x}{\sqrt{a^2-x^2}}$



$\therefore \sin\theta = \frac{x}{a}$

$\Rightarrow \theta = \sin^{-1}\left(\frac{x}{a}\right)$

6 **(c)**

$\because T_r = \sin^{-1}\left(\frac{\sqrt{r}-\sqrt{(r-1)}}{\sqrt{r(r+1)}}\right)$

$= \tan^{-1}\left(\frac{\sqrt{r}-\sqrt{(r-1)}}{1+\sqrt{r}\sqrt{(r-1)}}\right)$

$S_n = \sum_{r=1}^{n}\tan^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{1+\sqrt{r}\sqrt{r-1}}\right)$

$= \sum_{r=1}^{n}\{\tan^{-1}\sqrt{r} - \tan^{-1}\sqrt{(r-1)}\}$

$= \tan^{-1}\sqrt{n} - \tan^{-1}\sqrt{0}$

$= \tan^{-1}\sqrt{n} - 0$

$\therefore S_\infty = \tan^{-1}\infty = \frac{\pi}{2}$

7 **(c)**

We have,

$\theta_1 = \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{1}{3}$

$\Rightarrow \theta_1 = \frac{\pi}{2} - \cos^{-1}\frac{4}{5} + \frac{\pi}{2} - \cos^{-1}\frac{1}{3}$

$\Rightarrow \theta_1 = \pi - \left(\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{1}{3}\right)$

$\Rightarrow \theta_1 = \pi - \theta_2 \Rightarrow \theta_2 = \pi - \theta_1$

Also,

$\theta_1 = \sin^{-1}\frac{4}{5} + \sin^{-}\frac{1}{3}$

$\Rightarrow \theta_1 = \tan^{-1}\frac{4}{3} + \tan^{-1}\frac{1}{2\sqrt{2}}$

$\Rightarrow \theta_1 = \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{2\sqrt{2}}}{1-\frac{4}{3}\times\frac{1}{2\sqrt{2}}}\right) = \tan^{-1}\left(\frac{8\sqrt{2}+3}{6\sqrt{2}-4}\right) < \frac{\pi}{2}$

$\therefore \theta_2 = \pi - \theta_1 \Rightarrow \theta_2 > \frac{\pi}{2}$

Hence, $\theta_1 < \theta_2$

8 **(d)**

$\cos^{-1}x, \sin^{-1}x$ are real, if $-1 \le x \le 1$

But $\cos^{-1}x > \sin^{-1}x$

$\Rightarrow 2\cos^{-1}x > \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{4}$

$\therefore \cos(\cos^{-1}x) < \cos\frac{\pi}{4}$

$\Rightarrow x < \frac{1}{\sqrt{2}}$

The common value are $-1 \le x < \frac{1}{\sqrt{2}}$

9 **(a)**

Roots of equation $x^2 - 9x + 8 = 0$ are 1 and 8

Let $y = [\sin^2\alpha + \sin^4\alpha + \sin^6\alpha + \ldots\infty]\log_e 2$

$\Rightarrow y = \frac{\sin^2\alpha}{1-\sin^2\alpha}\log_e 2 = \tan^2\alpha \log_e 2$

$\Rightarrow y = \log_e 2^{\tan^2\alpha}$

$\Rightarrow e^y = 2^{\tan^2\alpha}$

According to question,

$2^{\tan^2\alpha} = 8 = 2^3 \Rightarrow \tan^2\alpha = 3$

$\Rightarrow \tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

$\therefore \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3} = \alpha$

10 **(c)**

Given, $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z + \cos^{-1}t = 4\pi$

Which is possible only when

$\cos^{-1}x = \cos^{-1}y = \cos^{-1}z = \cos^{-1}t = \pi$

[$\because$ Domain of $\cos^{-1}x$ is $[0, \pi]$]

$\Rightarrow x = y = z = t = \cos\pi = -1$

$\therefore x^2 + y^2 + z^2 + t^2$

$= (-1)^2 + (-1)^2 + (-1)^2 + (-1)^2$

$= 4$

11 **(b)**

Here, $T_n = \cot^{-1}\left(n^2 + \frac{3}{4}\right)$

$= \tan^{-1}\left(\frac{4}{4n^2+3}\right)$

$= \tan^{-1}\left(\frac{1}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right)$

$= \tan^{-1}\left[\frac{\left(n+\frac{1}{2}\right)-\left(n-\frac{1}{2}\right)}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right]$

$= \tan^{-1}\left(n+\frac{1}{2}\right) - \tan^{-1}\left(n-\frac{1}{2}\right)$

$\therefore S_\infty = T_\infty^{-1} - \tan^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow S_\infty = \cot^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow S_\infty = \tan^{-1}(2)$

12 **(c)**

Since, $-\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}$

$\therefore \sin^{-1}\alpha = \frac{\pi}{2}, \sin^{-1}\beta = \frac{\pi}{2}$ and $\sin^{-1}\gamma = \frac{\pi}{2}$

$\therefore \alpha = \beta = \gamma = 1$

Thus, $\alpha\beta + \alpha\gamma + \gamma\beta = 3$

13 **(a)**

$\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$

$= \frac{\pi}{4} + \pi + \tan^{-1}\left(\frac{2+3}{1-2\cdot 3}\right)$ (as $2\cdot 3 > 1$)

$= \frac{5\pi}{4} + \tan^{-1}(-1) = \frac{5\pi}{4} - \frac{\pi}{4} = \pi$

14 **(b)**

$\cos^{-1}p + \cos^{-1}q + \cos^{-1}r = 3\pi$

We know that, if $y = \cos^{-1}x$, then $-1 \le x \le 1$ and $0 \le y \le \pi$,

Hence, the given equation will hold only when each is π

$\therefore p = q = r = \cos\pi = -1$

$\therefore p^2 + q^2 + r^2 + 2pqr$

$= (-1)^2 + (-1)^2 + (-1)^2 + 2(-1)(-1)(-1)$

$= 1 + 1 + 1 - 2$

$= 3 - 2 = 1$

15 **(d)**

We have, $\theta = \sin^{-1}x + \cos^{-1}x - \tan^{-1}x$

$= \frac{\pi}{2} - \tan^{-1}x = \cot^{-1}x$

Since, $0 \le x \le 1$, therefore $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

16 **(d)**

We have,

$\cot\left\{\cos^{-1}\left(\frac{7}{25}\right)\right\} = \cot\left\{\cot^{-1}\left(\frac{7}{24}\right)\right\} = \frac{7}{24}$

17 **(b)**

Let $\tan^{-1}x = \theta$. Then, $x = \tan\theta$

Also,

$x \in (1, \infty) \Rightarrow \tan\theta > 1 \Rightarrow \frac{\pi}{4} < \theta < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < 2\theta < \pi$

Now,

$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= \sin^{-1}(\sin(\pi - 2\theta))$

$= \pi - 2\theta \quad \left[\because \frac{\pi}{4} < \theta < \frac{\pi}{2} \Rightarrow -\frac{\pi}{2} < \pi - 2\theta < 0\right]$

$= \pi - 2\tan^{-1}x$

19 **(a)**

$\tan\left\{\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right\}$

$= \tan\left\{\pi - \cos^{-1}\left(\frac{2}{7}\right) - \frac{\pi}{2}\right\}$

$= \tan\left\{\frac{\pi}{2} - \cos^{-1}\left(\frac{2}{7}\right)\right\} = \tan\left\{\sin^{-1}\left(\frac{2}{7}\right)\right\}$

$= \tan\left\{\tan^{-1}\left(\frac{3}{3\sqrt{5}}\right)\right\} = \frac{2}{3\sqrt{5}}$

20 **(c)**

Since, $\tan^{-1} x$ and $\cot^{-1} x$ exists for all $x \in R$ and $\cos^{-1}(2-x)$ exists, if $-1 \le 2-x \le 1$

$\therefore \tan^{-1} x - \cot^{-1} x = \cos^{-1}(2-x)$

Is possible only if $1 \le x \le 3$.

Thus the solution of given equation is [1, 3].

21 **(a)**

$\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$

$= \frac{\pi}{4} + \pi + \tan^{-1}\left(\frac{2+3}{1-2\cdot3}\right)$ (as $2\cdot3>1$)

$= \frac{5\pi}{4} + \tan^{-1}(-1) = \frac{5\pi}{4} - \frac{\pi}{4} = \pi$

22 **(b)**

Let $a = b\cos\theta$. Then,

$$a_1 = \frac{b\cos\theta + b}{2} = b\cos^2\frac{\theta}{2}$$

$$\Rightarrow b_1 = \sqrt{b\cos^2\frac{\theta}{2}\, b} = b\cos\frac{\theta}{2}$$

Now,

$$a_2 = \frac{a_1 + b_1}{2}$$

$$\Rightarrow a_2 = \frac{b\cos^2\frac{\theta}{2} + b\cos\frac{\theta}{2}}{2}$$

$$\Rightarrow a_2 = b\cos\frac{\theta}{2}\cos^2\frac{\theta}{4}$$

$$\Rightarrow b_2 = \sqrt{a_2 b_1} = \sqrt{b\cos\frac{\theta}{2}\cos^2\frac{\theta}{4}\, b\cos\frac{\theta}{2}}$$

$$\Rightarrow b_2 = b\cos\frac{\theta}{2}\cos\frac{\theta}{2^2}$$

Thus, $b_2 = b\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2^2}\right)$

Similarly, we have

$$b_3 = b\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2^2}\right)\cos\left(\frac{\theta}{2^3}\right)$$

and, so on

$$b_n = b\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2^2}\right)\cos\left(\frac{\theta}{2^3}\right)\ldots\cos\left(\frac{\theta}{2^n}\right)$$

Now,

$$b_\infty = \lim_{n\to\infty} b_n$$

$$= \lim_{n\to\infty} b\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2^2}\right)\cos\left(\frac{\theta}{2^3}\right)\ldots\cos\left(\frac{\theta}{2^n}\right)$$

$$\Rightarrow b_\infty = \lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{b\sin\theta}{2^n\sin\left(\frac{\theta}{2^n}\right)}$$

$$\Rightarrow b_\infty = \lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{\left(\frac{\theta}{2^n}\right)b\sin\theta}{\sin\left(\frac{\theta}{2^n}\right)\theta}$$

$$\Rightarrow b_\infty = \lim_{n\to\infty} b_n = \frac{b\sin\theta}{\theta} = \frac{b\sqrt{1-\frac{a^2}{b^2}}}{\cos^{-1}\left(\frac{a}{b}\right)} = \frac{\sqrt{b^2-a^2}}{\cos^{-1}\left(\frac{a}{b}\right)}$$

23 **(c)**

$$\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \cos\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$= \cos\cos^{-1}\sqrt{1-\frac{3}{4}}$$

$$= \cos\cos^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}$$

24 **(b)**

We have,

$\sin[\cot^{-1}\{\cos(\tan^{-1} x)\}]$

$$= \sin\left[\cot^{-1}\left\{\frac{1}{\sqrt{1+\tan^2(\tan^{-1} x)}}\right\}\right]$$

$$= -\sin\left\{\cot^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right)\right\}$$

$$= \frac{1}{\sqrt{1+\cot^2\left\{\cot^{-1}\frac{1}{\sqrt{1+x^2}}\right\}}} = \frac{1}{\sqrt{1+\frac{1}{1+x^2}}}$$

$$= \sqrt{\frac{1+x^2}{2+x^2}}$$

25 **(a)**

$\cot^{-1}(2.1^2) + \cot^{-1}(2.2^2) + \cot^{-1}(2.3^2) + \cdots\infty$

$$= \sum_{r=1}^{\infty}\cot^{-1}(2.r^2)$$

$$= \sum_{r=1}^{\infty}\tan^{-1}\left(\frac{1}{2r^2}\right)$$

$$= \sum_{r=1}^{\infty}\tan^{-1}\left(\frac{(1+2r)+(1-2r)}{1-(1+2r)(1-2r)}\right)$$

$$= \sum_{r=1}^{\infty}[\tan^{-1}(1+2r) + \tan^{-1}(1-2r)]$$

$= \tan^{-1}3 - \tan^{-1}1$

$+ \tan^{-1}5$

$- \tan^{-1}3$

$+ \tan^{-1}7 - \tan^{-1}5 + \ldots + \tan^{-1}\infty$

$$= -\frac{\pi}{4} + \frac{\pi}{2} = \frac{\pi}{4}$$

26 **(b)**

Let $\tan^{-1} x = \theta$. Then, $x = \tan\theta$

Also,

$x \in (1,\infty) \Rightarrow 1 < x < \infty \Rightarrow 1 < \tan\theta < \infty \Rightarrow \frac{\pi}{4} < \theta < \frac{\pi}{2}$

Now,

$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}(\tan 2\theta)$
$= \tan^{-1}(-\tan(\pi - 2\theta))$
$= \tan^{-1}(\tan(2\theta - \pi))$
$= 2\theta - \pi \left[\begin{array}{l} \because \frac{\pi}{4} < \theta < \frac{\pi}{2} \Rightarrow < 2\theta < \pi \\ \Rightarrow -\frac{\pi}{2} - 2\theta - \pi < 0 \end{array}\right]$
$= 2\tan^{-1} x - \pi$

27 **(d)**
$\cos(2\cos^{-1} x + \sin^{-1} x)$
$= \cos[2(\cos^{-1} x + \sin^{-1} x) - \sin^{-1} x]$
$= \cos(\pi - \sin^{-1} x) = -\cos(\sin^{-1} x)$
$= -\cos\left[\sin^{-1}\left(-\frac{1}{5}\right)\right] \quad \left(\because x = \frac{1}{5}\right)$
$= -\cos\left(\cos^{-1}\frac{2\sqrt{6}}{5}\right)$
$= -\frac{2\sqrt{6}}{5}$

28 **(a)**
$\cot^{-1}\frac{xy+1}{x-y} + \cot^{-1}\frac{yz+1}{y-z} + \cot^{-1}\frac{zx+1}{z-x}$
$= \cot^{-1} y - \cos^{-1} x$
$+ \cot^{-1} z$
$- \cot^{-1} y + \cot^{-1} x - \cot^{-1} z$
$= 0$

29 **(d)**
$\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$
$= \tan(\tan^{-1} 7 - \tan^{-1} 4)$
$= \tan\left[\tan^{-1}\left(\frac{7-4}{1+28}\right)\right] = \frac{3}{29}$

31 **(c)**
Given that, $\angle A = \tan^{-1} 2, \angle B = \tan^{-1} 3$
We know that, $\angle A + \angle B + \angle C = \pi$
$\Rightarrow \tan^{-1} 2 + \tan^{-1} 3 + \angle C = \pi$
$\Rightarrow \tan^{-1}\left(\frac{2+3}{1-2\times 3}\right) + \angle C = \pi$
$\Rightarrow \tan^{-1}(-1) + \angle C = \pi$
$\Rightarrow \frac{3\pi}{4} + \angle C = \pi$
$\Rightarrow \angle C = \frac{\pi}{4}$

32 **(a)**
Since, $0 \le \cos^{-1}\left(\frac{x^2}{2} + \sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right) \le \frac{\pi}{2}$
Because $\cos^{-1} x$ is in first quadrant when x is positive
And $\cos^{-1}\frac{x}{2} - \cos^{-1} x \ge 0$
So, $\cos^{-1}\frac{x}{2} \ge \cos^{-1} x$
Also, $\left|\frac{x}{2}\right| \le 1, |x| \le 1 \Rightarrow |x| \le 1$

33 **(c)**
$8x^2 + 22x + 5 = 0 \Rightarrow x = -\frac{1}{4}, -\frac{5}{2}$
$\therefore -1 < -\frac{1}{4} < 1$ and $-\frac{5}{2} < -1$
$\therefore \sin^{-1}\left(-\frac{1}{4}\right)$ exists but $\sin^{-1}\left(-\frac{5}{2}\right)$ does not exist.
$\sec^{-1}\left(-\frac{5}{2}\right)$ exists but $\sec^{-1}\left(-\frac{1}{4}\right)$does not exist.
$\tan^{-1}\left(-\frac{1}{4}\right)$ and $\tan^{-1}\left(-\frac{5}{2}\right)$ both exist.

34 **(d)**
Given, $(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$
$\therefore \quad (\tan^{-1} x + \cot^{-1} x)^2 - 2\tan^{-1} x\left(\frac{\pi}{2} - \tan^{-1} x\right)$
$= \frac{5\pi^2}{8}$
$\Rightarrow \quad \frac{\pi^2}{4} - 2 \times \frac{\pi}{2} = \tan^{-1} x + 2(\tan^{-1} x)^2 \frac{5\pi^2}{8}$
$\Rightarrow 2(\tan^{-1} x)^2 - \pi\tan^{-1} x - \frac{3\pi^2}{8} = 0$
$\Rightarrow \tan^{-1} x = -\frac{\pi}{4}, \frac{3\pi}{4}$
Now, we take $\tan^{-1} x = -\frac{\pi}{4} \Rightarrow x = -1$

35 **(a)**
We have, $\sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$
$= \sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{1+(m^2+m+1)(m^2-m+1)}\right)$
$= \sum_{m=1}^{n} \tan^{-1}\left(\frac{(m^2+m+1)-(m^2-m+1)}{1+(m^2+m+1)(m^2-m+1)}\right)$
$= \sum_{m=1}^{n} [\tan^{-1}(m^2+m+1) - \tan^{-1}(m^2-m+1)]$
$= (\tan^{-1} 3$
$- \tan^{-1} 1) + (\tan^{-1} 7 - \tan^{-1} 3) + (\tan^{-1} 13 - \tan^{-1}$
$+ n + 1) - \tan^{-1}(n^2 - n + 1)]$
$= \tan^{-1}\frac{n^2+n+1-1}{1+(n^2+n+1)\cdot 1}$
$= \tan^{-1}\left(\frac{n^2+n}{2+n^2+n}\right)$

36 **(d)**
$\tan\left(\cos^{-1}\frac{1}{5\sqrt{2}} - \sin^{-1}\frac{4}{\sqrt{17}}\right)$
$= \tan(\tan^{-1} 7 - \tan^{-1} 4)$
$= \tan\left[\tan^{-1}\left(\frac{7-4}{1+28}\right)\right] = \frac{3}{29}$

37 **(a)**
As we know that
$|\sin^{-1} x| \le \frac{\pi}{2}$

∴ Given relation

$\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$

Is possible only when

$\sin^{-1}x+\sin^{-1}y=\sin^{-1}z=\frac{\pi}{2}$

$\Rightarrow x=y=z=1$

$\therefore x^{100}+y^{100}+z^{100}-\frac{9}{x^{101}+y^{101}+z^{101}}$

$=1+1+1-\frac{9}{1+1+1}$

$=3-\frac{9}{3}=0$

39 **(d)**

$\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}=\tan^{-1}\frac{2x}{1-x^2}$

$\Rightarrow 2\tan^{-1}a-2\tan^{-1}b=2\tan^{-1}x$

$\Rightarrow \tan^{-1}\frac{a-b}{1+ab}=\tan^{-1}x$

$\Rightarrow x=\frac{a-b}{1+ab}$

40 **(a)**

$\because \tan^{-1}\left(\frac{1}{1+r+r^2}\right)=\tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)$

$=\tan^{-1}(r+1)-\tan^{-1}(r)$

$\therefore \sum_{r=0}^{n}[\tan^{-1}(r+1)-\tan^{-1}(r)]$

$=\tan^{-1}(n+1)-\tan^{-1}(0)$

$=\tan^{-1}(n+1)$

$\Rightarrow \sum_{r=0}^{\infty}\tan^{-1}\left(\frac{1}{1+r+r^2}\right)=\tan^{-1}(\infty)=\frac{\pi}{2}$

41 **(d)**

$4\tan^{-1}\frac{1}{5}=2\left[2\tan^{-1}\frac{1}{5}\right]$

$=2\tan^{-1}\frac{\frac{2}{5}}{1-\frac{1}{25}}=2\tan^{-1}\frac{5}{12}$

$=\tan^{-1}\frac{\frac{10}{12}}{1-\frac{25}{144}}$

$=\tan^{-1}\frac{120}{119}$

So, $4\tan^{-1}\frac{1}{5}-\tan^{-1}\frac{1}{239}=\tan^{-1}\frac{120}{119}-\tan^{-1}\frac{1}{239}$

$=\tan^{-1}\frac{\frac{120}{119}-\frac{1}{239}}{1+\frac{120}{119}\times\frac{1}{239}}$

$=\tan^{-1}\frac{(120\times 239)-119}{(119\times 239)+120}$

$=\tan^{-1}\frac{28561}{28561}=\tan^{-1}1=\frac{\pi}{4}$

42 **(c)**

The given expression can be written as

$\tan^{-1}\left\{a\sqrt{\frac{a+b+c}{abc}}\right\}+\tan^{-1}\left\{b\sqrt{\frac{a+b+c}{abc}}\right\}$

$+\tan^{-1}\left\{c\sqrt{\frac{a+b+c}{abc}}\right\}$

$=\tan^{-1}(ay)+\tan^{-1}(by)+\tan^{-1}(cy)$, where $y=\sqrt{\frac{a+b+c}{abc}}$

$=\tan^{-1}\left\{\frac{ay+by+cy-abc\,y^3}{1-ab\,y^2-bc\,y^2-ac\,y^2}\right\}$

$=\tan^{-1}\left\{y\left(\frac{a+b+c-abc\,y^2}{1-y^2(ab+bc+ca)}\right)\right\}=\tan^{-1}0$

$=0$

43 **(a)**

Given, $\sec^{-1}\sqrt{1+x^2}+\operatorname{cosec}^{-1}\frac{\sqrt{1+y^2}}{y}+\cot^{-1}\frac{1}{z}=\pi$

$\therefore \tan^{-1}x+\tan^{-1}y+\tan^{-1}z=\pi$

$\Rightarrow \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right)=\pi$

$\Rightarrow x+y+z=xyz$

44 **(d)**

Given, $\tan^{-1}(x-1)+\tan^{-1}x=\tan^{-1}3x-\tan^{-1}(x+1)$

$\Rightarrow \tan^{-1}\left[\frac{(x-1)+x}{1-(x-1)x}\right]=\tan^{-1}\left[\frac{3x-(x+1)}{1+3x(x+1)}\right]$

$\Rightarrow (1+3x^2+3x)(2x-1)$

$=(1-x^2+x)(2x-1)$

$\Rightarrow (2x-1)(4x^2+2x)=0$

$\Rightarrow x=0,\pm\frac{1}{2}$

45 **(a)**

As we know that

$|\sin^{-1}x|\le\frac{\pi}{2}$

∴ Given relation

$\sin^{-1}x+\sin^{-1}y+\sin^{-1}z=\frac{3\pi}{2}$

Is possible only when

$\sin^{-1}x+\sin^{-1}y=\sin^{-1}z=\frac{\pi}{2}$

$\Rightarrow x=y=z=1$

$\therefore x^{100}+y^{100}+z^{100}-\frac{9}{x^{101}+y^{101}+z^{101}}$

$=1+1+1-\frac{9}{1+1+1}$

$=3-\frac{9}{3}=0$

46 **(a)**

$$\sin^{-1}x+\sin^{-1}\frac{1}{x}+\cos^{-1}x+\cos^{-1}\frac{1}{x}$$
$$=[\sin^{-1}x+\cos^{-1}x]+\left[\sin^{-1}\left(\frac{1}{x}\right)+\cos^{-1}\left(\frac{1}{x}\right)\right]$$
$$=\frac{\pi}{2}+\frac{\pi}{2}=\pi$$

47 **(a)**

$$\sum_{m=1}^{n}\tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$$
$$=\sum_{m=1}^{n}\tan^{-1}\left(\frac{(m^2+m+1)-(m^2-m+1)}{1+(m^2+m+1)(m^2-m+1)}\right)$$
$$=\sum_{m=1}^{n}[\tan^{-1}(m^2+m+1)-\tan^{-1}(m^2-m+1)]$$
$$=\tan^{-1}(n^2+n+1)-\tan^{-1}1$$
$$=\tan^{-1}\left(\frac{n^2+n}{2+n^2+n}\right)$$

48 **(a)**

Let $\sin^{-1}a=A,\sin^{-1}b=B,\sin^{-1}c=C$

$\therefore \sin A=a,\sin B=b,\sin C=c$ ... (i)

And $A+B+C=\pi$

Then $\sin 2A+\sin 2B+\sin 2C=$

$4\sin A\sin B\sin C$ ... (ii)

$\Rightarrow \sin A\cos A+\sin B\cos B+\sin C\cos C$
$=2\sin A\sin B\sin C$

$\Rightarrow \sin A\sqrt{(1-\sin^2 A)}+\sin B\sqrt{(1-\sin^2 B)}$
$+\sin C\sqrt{1-\sin^2 C}=2\sin A\sin B\sin C$ ... (iii)

$\Rightarrow a\sqrt{(1-a^2)}+b\sqrt{(1-b^2)}+c\sqrt{(1-c^2)}$
$=2abc$

49 **(a)**

1 rad > 45°

$\Rightarrow \tan 1°>\tan 45°\Rightarrow \tan 1>1$

Also, $\tan^{-1}1=\frac{\pi}{4}<1$

Hence, $\tan 1>\tan^{-1}1$

50 **(d)**

Since, $2\cos^{-1}x=\cos^{-1}(2x^2-1)$

Therefore,

$2\cos^{-1}0.8=\cos^{-1}(2\times 0.64-1)=\cos^{-1}(0.28)$

$\Rightarrow \cos(2\cos^{-1}0.8)=\cos(\cos^{-1}0.28)=0.28$

51 **(a)**

$$\tan^{-1}2x+\tan^{-1}3x=\frac{\pi}{4}$$
$$\Rightarrow \frac{3x+2x}{1-6x^2}=\frac{\pi}{4}$$
$\Rightarrow 5x=1-6x^2$

$\Rightarrow 6x^2+5x-1=0$

$\Rightarrow x=-1,\frac{1}{6}$

But when $x=-1$,

$\tan^{-1}2x=\tan^{-1}(-2)<0$

And $\tan^{-1}3x=\tan^{-1}(-3)<0$

This value will not satisfy the given equation

Hence, $x=\frac{1}{6}$

52 **(c)**

We have,

$$\cos\left[\frac{1}{2}\cos^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{63}}{8}\right)\right\}\right]$$
$$=\cos\left[\frac{1}{2}\cos^{-1}\left\{\cos\left(\cos^{-1}\frac{1}{8}\right)\right\}\right]$$
$$=\cos\left[\frac{1}{2}\cos^{-1}\left(\frac{1}{8}\right)\right]=\sqrt{\frac{1+\cos\left(\cos^{-1}\frac{1}{8}\right)}{2}}=\frac{3}{4}$$

53 **(b)**

Let $\cos^{-1}x=\theta$. Then, $x=\cos\theta$

Also, $0\le x\le 1\Rightarrow 0\le\cos\theta\le 1\Rightarrow 0\le\theta\le\frac{\pi}{2}$

Now,

$\cos^{-1}(2x^2-1)$

$=\cos^{-1}(2\cos^2\theta-1)$

$=\cos^{-1}(\cos 2\theta)=2\theta=2\cos^{-1}x\ [\because 0\le 2\theta\le\pi]$

54 **(d)**

Let $\cot^{-1}x=\theta\Rightarrow x=\cot\theta$

Now, $\operatorname{cosec}\theta=\sqrt{1+\cot^2\theta}=\sqrt{1+x^2}$

$$\Rightarrow \sin\theta=\frac{1}{\operatorname{cosec}\theta}=\frac{1}{\sqrt{1+x^2}}$$
$$\Rightarrow \theta=\sin^{-1}\frac{1}{\sqrt{1+x^2}}$$
$$\therefore \sin(\cot^{-1}x)=\sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)$$
$$=\frac{1}{\sqrt{1+x^2}}=(1+x^2)^{-1/2}$$

55 **(b)**

$$\frac{\sin 2-1}{\cos 2}=-\frac{1-\sin 2}{\cos 2}$$
$$=-\frac{(\cos 1-\sin 1)^2}{(\cos 1+\sin 1)(\cos 1-\sin 1)}$$
$$=-\frac{\cos 1-\sin 1}{\cos 1+\sin 1}$$
$$=-\frac{1-\tan 1}{1+\tan 1}$$
$$=-\tan\left(\frac{\pi}{4}-1\right)$$
$$=\tan\left(1-\frac{\pi}{4}\right)$$
$$\Rightarrow \tan^{-1}\left(\frac{\sin 2-1}{\cos 2}\right)$$
$$=\tan^{-1}\left[\tan\left(1-\frac{\pi}{4}\right)\right]$$

$= 1 - \frac{\pi}{4}$

56 **(c)**

$\because$ ΔABC is right angled at A.

$\therefore \quad a^2 = b^2 + c^2 \quad$ ...(i)

Now, $\tan^{-1}\left(\frac{c}{a+b}\right) + \tan^{-1}\left(\frac{b}{a+c}\right)$

$= \tan^{-1}\left[\frac{\frac{c}{a+b} + \frac{b}{a+c}}{1 - \left(\frac{c}{a+b}\right)\left(\frac{b}{a+c}\right)}\right]$

$= \tan^{-1}\left[\frac{ac + c^2 + ab + b^2}{a^2 + ac + ab + bc - bc}\right]$

$= \tan^{-1}\left[\frac{a^2 + ac + ab}{a^2 + ac + ab}\right]$

$= \tan^{-1}(1) = \frac{\pi}{4} \quad$ [using Eq. (i)]

57 **(b)**

Let $\cos^{-1} x = \theta$. Then, $x = \cos\theta$

Also,

$-\frac{1}{2} \le x \le \frac{1}{2} \Rightarrow -\frac{1}{2} \le \cos\theta \le \frac{1}{2} \Rightarrow \frac{\pi}{3} \le \theta \le \frac{2\pi}{3}$

Now,

$\cos^{-1}(4x^3 - 3x) = \cos^{-1}(\cos 3\theta)$

$= \cos^{-1}(\cos(2\pi - 3\theta))$

$= 2\pi - 3\theta \quad \left[\because \frac{\pi}{3} \le \theta \le \frac{2\pi}{3} \Rightarrow 0 \le 2\pi - 3\theta \le \pi\right]$

$= 2\pi - 3\cos^{-1} x$

58 **(a)**

Let $\sin^{-1} a = A, \sin^{-1} b = B, \sin^{-1} c = C$

$\therefore \sin A = a, \sin B = b, \sin C = c$ ... (i)

And $A + B + C = \pi$

Then $\sin 2A + \sin 2B + \sin 2C =$

$4 \sin A \sin B \sin C$ ... (ii)

$\Rightarrow \sin A \cos A + \sin B \cos B + \sin C \cos C$

$= 2 \sin A \sin B \sin C$

$\Rightarrow \sin A\sqrt{(1 - \sin^2 A)} + \sin B\sqrt{(1 - \sin^2 B)}$

$+ \sin C\sqrt{1 - \sin^2 C} = 2 \sin A \sin B \sin C$ ... (iii)

$\Rightarrow a\sqrt{(1 - a^2)} + b\sqrt{(1 - b^2)} + c\sqrt{(1 - c^2)}$

$= 2abc$

59 **(c)**

Given, $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x$

$\Rightarrow \quad \sin^{-1}(1 - x) = \frac{\pi}{2} - \sin^{-1} x - \sin^{-1} x$

$= \frac{\pi}{2} - 2\sin^{-1} x$

$\Rightarrow \sin^{-1}(1 - x) = \sin^{-1} 1 - \sin^{-1} 2x\sqrt{1 - x^2}$

$\Rightarrow \sin^{-1}(1 - x) = \sin^{-1}[1\sqrt{1 - 4x^2(1 - x^2)} - 0]$

$\Rightarrow \quad (1 - x) = \sqrt{1 - 4x^2 + 4x^4}$

$\Rightarrow \quad 1 - x = 1 - 2x^2$

$\Rightarrow \quad 2x^2 - x = 0$

$\Rightarrow \quad x \in \left\{0, \frac{1}{2}\right\}$

60 **(c)**

Given, $\tan^{-1} a + \tan^{-1} b + \tan^{-1} c = \sin^{-1} 1$

$\therefore \tan^{-1}\left(\frac{a + b + c - abc}{1 - ab - bc - ca}\right) = \frac{\pi}{2}$

$\Rightarrow \frac{a + b + c - abc}{1 - ab - bc - ca} = \frac{1}{0} \Rightarrow ab + bc + ca - 1$

$= 0$

$\Rightarrow \quad \frac{1}{a} + \frac{1}{b} + \frac{1}{c} - \frac{1}{abc} = 0$

61 **(d)**

Now, $\cos^{-1}(\cos 4) = \cos^{-1}\{\cos(2\pi - 4)\} = 2\pi - 4$

$\Rightarrow 2\pi - 4 > 3x^2 - 4x$

$\Rightarrow 3x^2 - 4x - (2\pi - 4) < 0$

$\Rightarrow \frac{2 - \sqrt{6\pi - 8}}{3} < x < \frac{2 + \sqrt{6\pi - 8}}{3}$

62 **(a)**

Let $x = -y, y > 0$

$\therefore \sin^{-1} x = \sin^{-1}(-y)$

$= -\sin^{-1} y$

$= -\cos^{-1}\sqrt{1 - y^2}$

$= -\cos^{-1}\sqrt{1 - x^2}$

63 **(c)**

Now, $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\frac{7}{8}$

$= \tan^{-1}\left(\frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2}.\frac{1}{3}}\right) + \tan^{-1}\frac{7}{8}$

$= \tan^{-1}\left(\frac{\frac{5}{6}}{\frac{5}{6}}\right) + \tan^{-1}\frac{7}{8}$

$= \tan^{-1}(1) + \tan^{-1}\frac{7}{8}$

$= \tan^{-1}\left(\frac{1 + \frac{7}{8}}{1 - \frac{7}{8}}\right)$

$= \tan^{-1}(15)$

64 **(b)**

We have,

$\tan^{-1}\left(\frac{1 - x}{1 + x}\right) = \tan^{-1} 1 - \tan^{-1} x = \frac{\pi}{4} - \tan^{-1} x$

We have, $0 \le x \le 1$

$\therefore 0 \le -\tan^{-1} x \le -\frac{\pi}{4}$

$\Rightarrow 0 \ge -\tan^{-1} x \ge -\frac{\pi}{4}$

$\Rightarrow \frac{\pi}{4} \ge \frac{\pi}{4} - \tan^{-1} x \ge 0 \Rightarrow \frac{\pi}{4} \ge \tan^{-1}\left(\frac{1 - x}{1 + x}\right) \ge 0$

65 **(d)**

Given, $\sin^{-1}\frac{2a}{1+a^2}-\cos^{-1}\frac{1-b^2}{1+b^2}$

$$=\tan^{-1}\frac{2x}{1-x^2}$$

$\Rightarrow\ 2\tan^{-1}a-2\tan^{-1}b=2\tan^{-1}x$

$\Rightarrow\ \tan^{-1}\left(\frac{a-b}{1+ab}\right)=\tan^{-1}x$

$\Rightarrow\ x=\frac{a-b}{1+ab}$

66 **(b)**

Given,

$\tan^{-1}\left(\frac{1}{2x+1}\right)+\tan^{-1}\left(\frac{1}{4x+1}\right)$

$$=\tan^{-1}\left(\frac{2}{x^2}\right)$$

$\Rightarrow\ \tan^{-1}\left(\frac{\frac{1}{2x+1}+\frac{1}{4x+1}}{1-\frac{1}{2x+1}\times\frac{1}{4x+1}}\right)$

$\Rightarrow\ \tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow\ \tan^{-1}\left(\frac{6x+2}{8x^2+6x}\right)=\tan^{-1}\left(\frac{2}{x^2}\right)$

$\Rightarrow\ \frac{6x+2}{8x^2+6x}=\frac{2}{x^2}$

$\Rightarrow\ 6x^3+2x^2=16x^2+12x$

$\Rightarrow\ 6x^3-14x^2-12x=0$

$\Rightarrow\ 2x(3x^2-7x-6)=0$

$\Rightarrow 2x(3x+2)(x-3)=0$

$\Rightarrow\ x=0,-\frac{2}{3},3$

But $x=-\frac{2}{3}$ does not satisfy the given relation

67 **(d)**

Let $\sin^{-1}x=\theta$. Then, $x=\sin\theta$

Also,

$-\frac{1}{2}\le x\le\frac{1}{2}\Rightarrow-\frac{1}{2}\le\sin\theta\le\frac{1}{2}\Rightarrow-\frac{\pi}{6}\le\theta\le\frac{\pi}{6}$

Now,

$\sin^{-1}(3x-4x^3)=\sin^{-1}(\sin 3\theta)$

$\Rightarrow\sin^{-1}(3x-4x^3)$

$$=3\theta\ \left[\because-\frac{\pi}{6}\le\theta\le\frac{\pi}{6}\Rightarrow-\frac{\pi}{2}\le 3\theta\le\frac{\pi}{2}\right]$$

$\Rightarrow\sin^{-1}(3x-4x^3)=3\sin^{-1}x$

68 **(c)**

Let $\tan^{-1}x=\theta$. Then, $x=\tan\theta$

Also,

$-\infty<x<-1\Rightarrow-\infty<\tan\theta<-1\Rightarrow-\frac{\pi}{2}<\theta<-\frac{\pi}{4}$

Now,

$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$=\sin^{-1}(\sin 2\theta)$

$=\sin^{-1}(-\sin(\pi+2\theta))$

$=\sin^{-1}(\sin(-\pi-2\theta))$

$=-\pi-2\theta\ \left[\because-\frac{\pi}{2}<\theta<-\frac{\pi}{4}\Rightarrow-\frac{\pi}{2}<-\pi-2\theta<0\right]$

$=-\pi-2\tan^{-1}x$

69 **(c)**

$\because T_r=\sin^{-1}\left(\frac{\sqrt{r}-\sqrt{(r-1)}}{\sqrt{r(r+1)}}\right)$

$=\tan^{-1}\left(\frac{\sqrt{r}-\sqrt{(r-1)}}{1+\sqrt{r}\sqrt{(r-1)}}\right)$

$S_n=\sum_{r=1}^{n}\tan^{-1}\left(\frac{\sqrt{r}-\sqrt{r-1}}{1+\sqrt{r}\sqrt{r-1}}\right)$

$=\sum_{r=1}^{n}\{\tan^{-1}\sqrt{r}-\tan^{-1}\sqrt{(r-1)}\}$

$=\tan^{-1}\sqrt{n}-\tan^{-1}\sqrt{0}$

$=\tan^{-1}\sqrt{n}-0$

$\therefore S_\infty=\tan^{-1}\infty=\frac{\pi}{2}$

70 **(c)**

Given, $\cos^{-1}x=\alpha$

$\Rightarrow\ x=\cos\alpha,\ 0<x<1$ ...(i)

Also, $\sin^{-1}(2x\sqrt{1-x^2})+\sec^{-1}\left(\frac{1}{2x^2-1}\right)=\frac{2\pi}{3}$

$\therefore\ \sin^{-1}\left(2\cos\alpha\sqrt{1-\cos^2\alpha}\right)$

$$+\sec^{-1}\left(\frac{1}{2\cos^2\alpha-1}\right)=\frac{2\pi}{3}$$

$\Rightarrow\ \sin^{-1}(\sin 2\alpha)+\sec^{-1}(\sec 2\alpha)=\frac{2\pi}{3}$

$\Rightarrow\ 2\alpha+2\alpha=\frac{2\pi}{3}$

l$\Rightarrow\ \alpha=\frac{\pi}{6}$

Now, $x=\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$

$\Rightarrow\ 2x=\sqrt{3}$

$\therefore\ \tan^{-1}(2x)=\tan^{-1}(\sqrt{3})=\frac{\pi}{3}$

71 **(a)**

Since, $\alpha=\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{1}{3}\right)$

$=\sin^{-1}\left(\frac{4}{5}\sqrt{1-\frac{1}{9}}+\frac{1}{3}\sqrt{1-\frac{16}{25}}\right)$

$\Rightarrow\alpha=\sin^{-1}\left(\frac{8\sqrt{2}}{15}+\frac{3}{15}\right)=\sin^{-1}\left(\frac{8\sqrt{2}+3}{15}\right)$

Since, $\frac{8\sqrt{2}+3}{15}<1$

$\therefore \alpha < \frac{\pi}{2}$

Now, $\beta = \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{1}{3}\right)$

$\Rightarrow \beta = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right) + \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{3}\right)$

$= \pi - \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{1}{3}\right)$

$= \pi - \alpha$

$\Rightarrow \beta > \alpha \quad \left(\because \alpha < \frac{\pi}{2}\right)$

72 **(c)**

Given that, $\theta = \tan^{-1} a$ and $\phi = \tan^{-1} b$

And $ab = -1$

$\therefore \tan\theta \tan\phi = ab = -1$

$\Rightarrow \tan\theta = -\cot\phi$

$\Rightarrow \tan\theta = \tan\left(\frac{\pi}{2} + \phi\right)$

$\Rightarrow \theta - \phi = \frac{\pi}{2}$

73 **(c)**

$\tan\theta + \tan\left(\frac{\pi}{3} + \theta\right) + \tan\left(-\frac{\pi}{3} + \theta\right) = a\tan 3\theta$

$\Rightarrow \tan\theta + \frac{\sqrt{3} + \tan\theta}{1 - \sqrt{3}\tan\theta} + \frac{\tan\theta - \sqrt{3}}{1 + \sqrt{3}\tan\theta} = a\tan 3\theta$

$\Rightarrow \tan\theta + \frac{8\tan\theta}{1 - 3\tan^2\theta} = a\tan 3\theta$

$\Rightarrow \frac{3(3\tan\theta - \tan^3\theta)}{1 - 3\tan^2\theta} = a\tan 3\theta$

$\Rightarrow 3\tan 3\theta = a\tan 3\theta$

$\Rightarrow a = 3$

74 **(b)**

Let $\cot^{-1}\frac{1}{2} = \phi \Rightarrow \frac{1}{2} = \cot\phi$

$\Rightarrow \sin\phi = \frac{1}{\sqrt{1 + \cot^2\phi}} = \frac{2}{\sqrt{5}}$

Let $\cos^{-1} x = \theta \Rightarrow \sec\theta = \frac{1}{x}$

$\Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1}$

$\Rightarrow \tan\theta = \sqrt{\frac{1}{x^2} - 1}$

$\Rightarrow \tan\theta = \frac{\sqrt{1 - x^2}}{x}$

Now, $\tan(\cos^{-1} x) = \sin\left(\cot^{-1}\frac{1}{2}\right)$

$\Rightarrow \tan\left(\tan^{-1}\frac{\sqrt{1 - x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$

$\Rightarrow \frac{\sqrt{1 - x^2}}{x} = \frac{2}{\sqrt{5}}$

$\Rightarrow \sqrt{(1 - x^2)5} = 2x$

On squaring both sides, we get

$(1 - x^2)5 = 4x^2$

$\Rightarrow 9x^2 = 5$

$\Rightarrow x = \pm\frac{\sqrt{5}}{3}$

75 **(a)**

We know, $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x = \frac{\pi}{2} - \frac{\pi}{5}$

$\Rightarrow \cos^{-1} x = \frac{3\pi}{10}$

76 **(b)**

Let $\sin^{-1} x = \theta$. Then, $x = \sin\theta$

Also,

$\frac{1}{2} \le x \le 1 \Rightarrow \frac{1}{2} \le \sin\theta \le 1 \Rightarrow \frac{\pi}{6} \le \theta \le \frac{\pi}{2} \Rightarrow \frac{\pi}{2}$

$\le 3\theta \le \frac{3\pi}{2}$

Now,

$\sin^{-1}(3x - 4x^3)$

$= \sin^{-1}(\sin 3\theta)$

$= \sin^{-1}(\sin(\pi - 3\theta))$

$= \pi - 3\theta \quad \left[\because \frac{\pi}{2} \le 3\theta \le \frac{3\pi}{2} \Rightarrow -\frac{\pi}{2} \le 3\theta \le \frac{\pi}{2}\right]$

$= \pi - 3\sin^{-1} x$

78 **(a)**

Since, x, y, z are in AP

$\therefore \quad y = \frac{x+z}{2} \qquad$ ...(i)

And $\tan^{-1} x, \tan^{-1} y$ and $\tan^{-1} z$ are also in AP.

$\therefore \quad 2\tan^{-1} y = \tan^{-1} x + \tan^{-1} z$

$\Rightarrow \tan^{-1}\left(\frac{2y}{1 - y^2}\right) = \tan^{-1}\left(\frac{x + z}{1 - xz}\right)$

$\Rightarrow \quad \frac{2y}{1-y^2} = \frac{2y}{1-xz} \qquad$ [from Eq. (i)]

$\Rightarrow \quad y^2 = xz$

$\Rightarrow$ x, y, z are in GP.

$\therefore \quad x = y = z$

79 **(c)**

Since, $a_1, a_2, a_3, \ldots, a_n$ are in AP with common difference 5

$\Rightarrow a_2 - a_1 = a_3 - a_2 = \cdots = a_n - a_{n-1} = 5$

Now $\quad T_1 = \tan^{-1}\frac{5}{1+a_1a_2}$

$= \tan^{-1}\frac{a_2 - a_1}{1 + a_2a_1}$

$= \tan^{-1} a_2 - \tan^{-1} a_1$

Similarly

$T_2 = \tan^{-1} a_3 - \tan^{-1} a_2$

$T_3 = \tan^{-1} a_4 - \tan^{-1} a_3$

$T_{n-1} = \tan^{-1} a_n - \tan^{-1} a_{n-1}$

On adding all, we get

$\therefore$ Required sum$= \tan^{-1} a_n - \tan^{-1} a_1$

$= \tan^{-1}\frac{a_n - a_1}{1 + a_na_1}$

$$= \tan^{-1}\frac{a_1 + 5(n-1) - a_1}{1 + a_n a_1}$$
$$= \tan^{-1}\frac{5(n-1)}{1 + a_n a_1}$$

80 **(b)**

Given, $\tan^{-1}\left(\frac{1+x}{1-x}\right) = \frac{\pi}{4} + \tan^{-1} x$

RHS= $\frac{\pi}{4} + \tan^{-1} x = \tan^{-1} 1 + \tan^{-1} x$

$= \tan^{-1}\left(\frac{1+x}{1-x}\right)$, if $x < 1$

$\therefore \quad x \in (-\infty, 1)$

81 **(d)**

We know that,

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$
$$= \frac{\frac{x\sqrt{3}}{2k-x} - \frac{2x-k}{k\sqrt{3}}}{1 + \frac{x\sqrt{3}}{2k-x}\cdot\frac{2x-k}{k\sqrt{3}}} = \frac{1}{\sqrt{3}}$$

$\Rightarrow A - B = 30°$

82 **(c)**

$$\sqrt{1+x^2}[\{x\cos(\cot^{-1}x) + \sin(\cot^{-1}x)\}^2 - 1]^{1/2}$$
$$= \sqrt{1+x^2}\left[\left\{x\cos\left(\cos^{-1}\frac{x}{\sqrt{1+x^2}}\right) + \sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)\right\}^2 - 1\right]^{1/2}$$
$$= \sqrt{1+x^2}\left[\left\{x.\frac{x}{\sqrt{1+x^2}} + \frac{1}{\sqrt{1+x^2}}\right\}^2 - 1\right]^{1/2}$$
$$= \sqrt{1+x^2}[1 + x^2 - 1]^{1/2}$$
$$= x\sqrt{1+x^2}$$

83 **(b)**

Since, $\sin^{-1}\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) + \sin^{-1}\left(\frac{4}{5}\right) = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \cos^{-1}\left(\frac{4}{5}\right)$

$\Rightarrow \sin^{-1}\left(\frac{x}{5}\right) = \sin^{-1}\left(\frac{3}{5}\right)$

$\Rightarrow \quad x = 3$

84 **(c)**

$\because -\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}, -\frac{\pi}{2} \le \sin^{-1} y \le \frac{\pi}{2}$

And $-\frac{\pi}{2} \le \sin^{-1} z \le \frac{\pi}{2}$

Given that, $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$

Which is possible only when

$\sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$

Or $x = y = z = 1$

Put $p = q = 1$

Then $f(2) = f(1)f(1) = 2 \cdot 2 = 4$

And put $p = 1, q = 2$

Then, $f(3) = f(1)f(2) = 2 \cdot 2^2 = 8$

$$\therefore x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$$
$$= 1 + 1 + 1 - \frac{3}{1+1+1}$$
$$= 3 - 1 = 2$$

86 **(d)**

$$\tan^{-1}(x+2) + \tan^{-1}(x-2) = \tan^{-1}\frac{1}{2}$$
$$\Rightarrow \tan^{-1}\frac{x+2+x-2}{1-(x+2)(x-2)} = \tan^{-1}\frac{1}{2}$$
$$\Rightarrow \frac{2x}{1-x^2+4} = \frac{1}{2}$$
$$\Rightarrow 4x = 5 - x^2$$
$$\Rightarrow x^2 + 4x - 5 = 0$$
$$\Rightarrow (x-1)(x+5) = 0$$
$$\Rightarrow x = 1, -5$$

87 **(b)**

$$\cos\left[\cos^{-1}\left(-\frac{1}{7}\right) + \sin^{-1}\left(-\frac{1}{7}\right)\right] = \cos\frac{\pi}{2}$$
$$\left[\because \cos^{-1} x = +\sin^{-1} x = \frac{\pi}{2}\right]$$
$$= 0$$

88 **(b)**

$$\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$$
$$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1} x\right) - \cos^{-1} x = \frac{\pi}{6}$$
$$\Rightarrow 2\cos^{-1} x = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$
$$\Rightarrow \cos^{-1} x = \frac{\pi}{6} \Rightarrow x = \frac{\sqrt{3}}{2}$$

89 **(b)**

$$\sec\left[\tan^{-1}\left(\frac{b+a}{b-a}\right) - \tan^{-1}\left(\frac{a}{b}\right)\right]$$
$$= \sec\left[\tan^{-1}\left\{\frac{\frac{b+a}{b-a} - \frac{a}{b}}{1 + \left(\frac{b+a}{b-a}\right)\left(\frac{a}{b}\right)}\right\}\right]$$
$$= \sec[\tan^{-1}(1)]$$
$$= \sec\frac{\pi}{4} = \sqrt{2}$$

90 **(a)**

$$\sin(\cos^{-1} x) = \cos(\sin^{-1} x)$$
$$\Rightarrow \sin\left(\frac{\pi}{2} - \sin^{-1} x\right) = \cos(\sin^{-1} x)$$
$$\Rightarrow \sin^{-1} x = \sin^{-1} x$$

91 **(d)**

$$\tan^{-1}\left(\frac{a}{b}\right) + \tan^{-1}\left(\frac{a+b}{a-b}\right)$$
$$= \tan^{-1}\left\{\frac{\frac{a}{b} + \frac{a+b}{a-b}}{1 - \frac{a}{b}\left(\frac{a+b}{a-b}\right)}\right\}$$

$$= \tan^{-1}\left(-\frac{a^2+b^2}{a^2+b^2}\right)$$

$$= \tan^{-1}(-1)$$

$\therefore$ The value is neither depends on a nor b

92 **(d)**

We have,

$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = \pi - 2\tan^{-1}x$ for $x \geq 1$

$$\therefore 2\tan^{-1}x + \sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x + \pi - 2\tan^{-1}x = \pi$$

93 **(c)**

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

$\Rightarrow \cos^{-1}\left(xy - \sqrt{1-y^2}\sqrt{1-x^2}\right) = \pi - \cos^{-1}z$

$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = \cos(\pi - \cos^{-1}z)$

$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

$\Rightarrow xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$

On squaring both sides, we get

$x^2y^2 + z^2 + 2xyz - 1 - x^2 - y^2 + x^2y^2$

$\Rightarrow x^2 + y^2 + z^2 = 1 - 2xyz$

94 **(b)**

Let $\tan^{-1}x = \theta$. Then, $x = \tan\theta$

Also,

$$x > \frac{1}{\sqrt{3}} \Rightarrow \tan\theta > \frac{1}{\sqrt{3}} \Rightarrow \frac{\pi}{6} < \theta < \frac{\pi}{2} \Rightarrow \frac{\pi}{2} < 3\theta < \frac{3\pi}{2}$$

Now,

$$\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \tan^{-1}(\tan 3\theta)$$

$$\Rightarrow \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \tan^{-1}(\tan(\pi - 3\theta))$$

$$\Rightarrow \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = \tan^{-1}(\tan(3\theta - \pi))$$

$$\Rightarrow \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = 3\theta - \pi \quad \left[\begin{array}{c}\because \frac{\pi}{6} < \theta < \frac{\pi}{2} \\ \Rightarrow -\frac{\pi}{2} < 3\theta - \pi < \frac{\pi}{2}\end{array}\right]$$

$$\Rightarrow \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = 3\tan^{-1}x - \pi$$

95 **(b)**

We have, $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\mathrm{cosec}\,x)$

$$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\,\mathrm{cosec}\,x)$$

$$\Rightarrow \frac{2\cos x}{\sin^2 x} = 2\,\mathrm{cosec}\,x$$

$$\Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$$

96 **(a)**

Let $\cot^{-1}\frac{3}{4} = \theta \Rightarrow \cot\theta = \frac{3}{4}$

And $\sin\theta = \frac{1}{\sqrt{1+\cot^2\theta}} = \frac{1}{\sqrt{1+\left(\frac{9}{16}\right)}} = \frac{4}{5}$

$$\therefore \cot^{-1}\frac{3}{4} + \sin^{-1}\frac{5}{13} = \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{5}{13}$$

$$= \sin^{-1}\left[\frac{4}{5}\sqrt{1-\frac{25}{269}} + \frac{5}{13}\sqrt{1-\frac{16}{25}}\right]$$

$$= \sin^{-1}\left[\frac{4}{5}\cdot\frac{12}{13} + \frac{5}{13}\cdot\frac{3}{5}\right]$$

$$= \sin^{-1}\left[\frac{48+15}{65}\right] = \sin^{-1}\frac{63}{65}$$

97 **(c)**

$\therefore \tan^{-1}x - \tan^{-1}y = 0 \Rightarrow x = y$

Also, $\cos^{-1}x + \cos^{-1}y = \frac{\pi}{2} \Rightarrow 2\cos^{-1}x = \frac{\pi}{2}$

$$\Rightarrow \cos^{-1}x = \frac{\pi}{4} \Rightarrow x = \frac{1}{\sqrt{2}} \Rightarrow x^2 = \frac{1}{2}$$

Hence, $x^2 + xy + y^2 = 3x^2 = \frac{3}{2}$

98 **(c)**

Clearly, $x(x+1) \geq 0$ and $x^2 + x + 1 \leq 1$

$\Rightarrow x(x+1) = 0$

$\Rightarrow x = 0, -1$

When $x = 0$,

LHS$= \tan^{-1}0 + \sin^{-1}1 = \frac{\pi}{2}$

When $x = -1$,

LHS$= \tan^{-1}0 + \sin^{-1}\sqrt{1-1+1}$

$$= 0 + \sin^{-1}(1) = \frac{\pi}{2}$$

Thus, the number of solution is 2

99 **(a)**

We have,

$\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$

$$= \tan^{-1}\left\{\frac{x+y+z-xyz}{1-(xy+yz+zx)}\right\} = \tan^{-1}0 = 0$$

100 **(a)**

Given, $\tan^{-1}x + \cos^{-1}\frac{y}{\sqrt{1+y^2}} = \sin^{-1}\frac{3}{\sqrt{10}}$

$$\Rightarrow \tan^{-1}x + \tan^{-1}\frac{1}{y} = \sin^{-1}\frac{3}{\sqrt{10}}$$

$$\Rightarrow \tan^{-1}\left(\frac{x+\frac{1}{y}}{1-\frac{x}{y}}\right) = \tan^{-1}3$$

$$\Rightarrow x + \frac{1}{y} = 3\left(1 - \frac{x}{y}\right)$$

$$\Rightarrow x = 1,\ y = 2$$

$\therefore$ The number of solutions of given equation is 1.

101 **(a)**

We have,

$\sin^{-1}\frac{3}{5}+\tan^{-1}\frac{1}{7}$

$=\tan^{-1}\frac{3}{4}+\tan^{-1}\frac{1}{7}$

$=\tan^{-1}\left\{\frac{3/4+1/7}{1-3/4\times 1/7}\right\}=\tan^{-1}\left(\frac{25}{25}\right)=\tan^{-1}1$

$=\frac{\pi}{4}$

103 **(b)**

Given that, $x^2+y^2+z^2=r^2$

Now, $\tan^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{yz}{xe}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)$

$=\tan^{-1}\left[\frac{\frac{xy}{zr}+\frac{yz}{xr}+\frac{xz}{yr}-\frac{xyz}{r^3}}{1-\left(\frac{x^2+y^2+z^2}{r^2}\right)}\right]$

$=\tan^{-1}\left[\frac{\frac{xy}{zr}+\frac{yz}{xr}+\frac{xz}{yr}-\frac{xyz}{r^3}}{1-\frac{r^2}{r^2}}\right]$

$=\tan^{-2}\infty=\frac{\pi}{2}$

104 **(b)**

Put $x=\sin\theta$, we get

$f(x)=\sin^{-1}\left\{\sin\left(\theta-\frac{\pi}{6}\right)\right\}$

For, $-\frac{1}{2}\le x\le 1$

$\Rightarrow\ -\frac{1}{2}\le\sin\theta\le 1$

$\Rightarrow\ -\frac{\pi}{6}\le\theta\le\frac{\pi}{2}$

$\Rightarrow\ \theta=\frac{\pi}{6}$

is in the fourth on the first quadrant

$\therefore\ f(x)=\theta-\frac{\pi}{6}=\sin^{-1}x-\frac{\pi}{6}$

105 **(c)**

$\cos^{-1}\left(\frac{1}{2}\right)+2\sin^{-1}\left(\frac{1}{2}\right)$

$=\frac{\pi}{3}+2\cdot\frac{\pi}{6}=\frac{\pi}{3}+\frac{\pi}{3}=\frac{2\pi}{3}$

106 **(c)**

Given, $\tan^{-1}2\theta+\tan^{-1}3\theta=\frac{\pi}{4}$

$\therefore\ \tan^{-1}\left(\frac{2\theta+3\theta}{1-2\theta\times 3\theta}\right)=\tan^{-1}1$

$\Rightarrow\ 6\theta^2+5\theta-1=0$

$\Rightarrow\ \theta=\frac{-5\pm\sqrt{25+24}}{2\times 6}$

$=\frac{-5\pm 7}{12}=-1,\frac{1}{6}$

$\Rightarrow\ \theta=\frac{1}{6}$

107 **(b)**

Let $\theta=\cos^{-1}\left(-\frac{1}{2}\right)$

$\Rightarrow\ \cos\theta=-\frac{1}{2}=-\cos\left(\frac{\pi}{3}\right)$

$=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right)$

$\Rightarrow\ \theta=2n\pi\pm\frac{2\pi}{3}$

$\Rightarrow\ \theta=\frac{4\pi}{3},\frac{8\pi}{3},\frac{10\pi}{3},\frac{14\pi}{3},\ldots$

108 **(c)**

$\tan\theta+\tan\left(\frac{\pi}{3}+\theta\right)+\tan\left(-\frac{\pi}{3}+\theta\right)=a\tan 3\theta$

$\Rightarrow\tan\theta+\frac{\sqrt{3}+\tan\theta}{1-\sqrt{3}\tan\theta}+\frac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta}=a\tan 3\theta$

$\Rightarrow\tan\theta+\frac{8\tan\theta}{1-3\tan^2\theta}=a\tan 3\theta$

$\Rightarrow\frac{3(3\tan\theta-\tan^3\theta)}{1-3\tan^2\theta}=a\tan 3\theta$

$\Rightarrow 3\tan 3\theta=a\tan 3\theta$

$\Rightarrow a=3$

109 **(a)**

Let $\cot^{-1}\frac{3}{4}=\theta\Rightarrow\cot\theta=\frac{3}{4}$

And $\sin\theta=\frac{1}{\sqrt{1+\cot^2\theta}}=\frac{1}{\sqrt{1+\left(\frac{9}{16}\right)}}=\frac{4}{5}$

$\therefore\cot^{-1}\frac{3}{4}+\sin^{-1}\frac{5}{13}=\sin^{-1}\frac{4}{5}+\sin^{-1}\frac{5}{13}$

$=\sin^{-1}\left[\frac{4}{5}\sqrt{1-\frac{25}{269}}+\frac{5}{13}\sqrt{1-\frac{16}{25}}\right]$

$=\sin^{-1}\left[\frac{4}{5}\cdot\frac{12}{13}+\frac{5}{13}\cdot\frac{3}{5}\right]$

$=\sin^{-1}\left[\frac{48+15}{65}\right]=\sin^{-1}\frac{63}{65}$

110 **(d)**

Now, $\cos^{-1}(\cos 4)=\cos^{-1}\{\cos(2\pi-4)\}=2\pi-4$

$\Rightarrow 2\pi-4>3x^2-4x$

$\Rightarrow 3x^2-4x-(2\pi-4)<0$

$\Rightarrow\frac{2-\sqrt{6\pi-8}}{3}<x<\frac{2+\sqrt{6\pi-8}}{3}$

111 **(c)**

Let $\tan^{-1}x=\theta$. Then, $x=\tan\theta$

Also,

$x\in(-\infty,-1)$

$\Rightarrow-\infty<x<1\Rightarrow-\infty<\tan\theta<-1\Rightarrow-\frac{\pi}{2}<\theta<-\frac{\pi}{4}$

Now,

$\tan^{-1}\left(\frac{2x}{1-x^2}\right)=\tan^{-1}(\tan 2\theta)$

$=\tan^{-1}(\tan(\pi+2\theta))$

$= \pi + 2\theta \quad \left[\because -\frac{\pi}{2} < \theta < -\frac{\pi}{4} \Rightarrow 0 < \pi + 2\theta < \frac{\pi}{2}\right]$

$= \pi + 2\tan^{-1} x$

112 **(b)**

Let $\sin^{-1} x = \theta$. Then, $x = \sin\theta$ and $\sqrt{1-x^2} = \cos\theta$

Also,

$\frac{1}{\sqrt{2}} \le x \le 1 \Rightarrow \frac{1}{\sqrt{2}} \le \sin\theta \le 1 \Rightarrow \frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

$\therefore \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= \sin^{-1}(\sin(\pi - 2\theta))$

$= \pi - 2\theta \quad \left[\because \frac{\pi}{4} \le \theta \le \frac{\pi}{2} \Rightarrow 0 \le \pi - 2\theta \le \frac{\pi}{2}\right]$

$= \pi - 2\sin^{-1} x$

114 **(a)**

Since, $-\frac{\pi}{2} < \sin^{-1} x \le \frac{\pi}{2}$

$\therefore \sin^{-1} x_i = \frac{\pi}{2}, 1 \le i \le 20$

$\Rightarrow x_i = 1, 1 \le i \le 20$

Thus, $\sum_{i=1}^{20} x_i = 20$

115 **(a)**

1 rad $> 45°$

$\Rightarrow \tan 1° > \tan 45° \Rightarrow \tan 1 > 1$

Also, $\tan^{-1} 1 = \frac{\pi}{4} < 1$

Hence, $\tan 1 > \tan^{-1} 1$

116 **(c)**

$\alpha + \beta = \sin^{-1}\frac{\sqrt{3}}{2} + \cos^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{1}{3} + \cos^{-1}\frac{1}{3}$

$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$

Also, $\alpha = \frac{\pi}{3} + \sin^{-1}\frac{1}{3} < \frac{\pi}{3} + \sin^{-1}\frac{1}{2}$

As $\sin\theta$ is increasing in $\left[0, \frac{\pi}{2}\right]$

$\therefore \quad \alpha < \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

$\Rightarrow \beta > \frac{\pi}{2} > \alpha$

$\Rightarrow \alpha < \beta$

117 **(d)**

$2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left[\frac{2\left(\frac{1}{3}\right)}{1-\frac{1}{9}}\right] + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right)$

$=\tan^{-1}\left(\frac{25}{25}\right) = \frac{\pi}{4}$

118 **(c)**

$\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$

$= \tan\left[\frac{1}{2}\cdot 2\tan^{-1} a + \frac{1}{2}\cdot 2\tan^{-1} a\right]$

$= \tan(2\tan^{-1} a)$

$= \tan\left[\tan^{-1}\left(\frac{2a}{1-a^2}\right)\right]$

$= \frac{2a}{1-a^2}$

119 **(c)**

Let $S_\infty = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \dots$

$\therefore T_n \cot^{-1} 2n^2$

$= \tan^{-1}\frac{1}{2n^2}$

$= \tan^{-1}\left(\frac{2}{4n^2}\right) = \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$

$\therefore S_n = \sum_{n=1}^{\infty}\{\tan^{-1}(2n+1) - \tan^{-1}(2n-1)\}$

$= \tan^{-1}\infty - \tan^{-1} 1$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

120 **(a)**

Given, $\tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}}\right) = \frac{\pi}{2}$

$\Rightarrow \frac{(a+b)x}{x^2-ab} = \tan\frac{\pi}{2}$

$\Rightarrow \frac{(a+b)x}{x^2-ab} = \frac{1}{0}$

$\Rightarrow x^2 - ab = 0$

$\Rightarrow x = \sqrt{ab}$

121 **(b)**

We have, $\Sigma x_1 = \sin 2\beta, \Sigma x_1x_2 = \cos 2\beta, \Sigma x_1x_2x_3 = \cos\beta$ and $x_1x_2x_3x_4 = -\sin\beta$

$\therefore \tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$

$= \tan^{-1}\left(\frac{\Sigma x_1 - \Sigma x_1x_2x_3}{1-\Sigma x_1x_2 + x_1x_2x_3x_4}\right)$

$= \tan^{-1}\left(\frac{\sin 2\beta - \cos\beta}{1-\cos 2\beta - \sin\beta}\right)$

$= \tan^{-1}\left(\frac{(2\sin\beta - 1)\cos\beta}{\sin\beta(2\sin\beta - 1)}\right)$

$= \tan^{-1}(\cot\beta)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \beta\right)\right) = \frac{\pi}{2} - \beta$

122 **(d)**

$\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\sin 2x}{5+3\cos 2x}\right)$

$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{\frac{6\tan x}{1+\tan^2 x}}{5+\frac{3(1-\tan^2 x)}{1+\tan^2 x}}\right)$

$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{6\tan x}{8+2\tan^2 x}\right)$

$=\tan^{-1}\left(\frac{\tan x}{4}\right)+\tan^{-1}\left(\frac{3\tan x}{4+\tan^2 x}\right)$

$=\tan^{-1}\left(\frac{\frac{\tan x}{4}+\frac{3\tan x}{4+\tan^2 x}}{1-\frac{3\tan^2 x}{4(4+\tan^2 x)}}\right)\left(\text{as }\left|\frac{\tan x}{4}\cdot\frac{3\tan x}{4\tan^2 x}\right|<1\right)$

$=\tan^{-1}\left(\frac{16\tan x+\tan^3 x}{16+\tan^2 x}\right)$

$=\tan^{-1}(\tan x)=x$

124 **(c)**

Let $\cos^{-1}x=\theta$. Then, $x=\cos\theta$

Also,

$-1\le x\le 0\Rightarrow -1\le\cos\theta\le 0\Rightarrow\frac{\pi}{2}\le\theta\le\pi$

Now,

$\cos^{-1}(2x^2-1)=\cos^{-1}(\cos 2\theta)$

$=\cos^{-1}(2\pi-2\theta)$

$=2\pi-2\theta\left[\because\frac{\pi}{2}\le\theta\le\pi\Rightarrow\pi\le 2\theta\le 2\pi\Rightarrow 0\le 2\pi-2\theta\le\pi\right]$

$=2\pi-2\cos^{-1}x$

125 **(b)**

$\therefore\cos^{-1}\frac{3}{5}-\sin^{-1}\frac{4}{5}=\cos^{-1}x$

$\Rightarrow\sin^{-1}\frac{4}{5}-\sin^{-1}\frac{4}{5}=\cos^{-1}x$

$\Rightarrow\cos^{-1}x=0\Rightarrow x=\cos 0=1$

$\therefore x=1$

126 **(d)**

We have,

$\sec^{-1}x=\operatorname{cosec}^{-1}y\Rightarrow\cos^{-1}\frac{1}{x}=\sin^{-1}\frac{1}{y}$

$\therefore\cos^{-1}\frac{1}{x}+\cos^{-1}\frac{1}{y}=\sin^{-1}\frac{1}{y}+\cos^{-1}\frac{1}{y}=\frac{\pi}{2}$

128 **(c)**

Let $\sin^{-1}x=\theta$. Then, $x=\sin\theta$

Also,

$-1\le x\le-\frac{1}{2}$

$\Rightarrow -1\le\sin\theta\le-\frac{1}{2}\Rightarrow-\frac{\pi}{2}\le\theta\le-\frac{\pi}{6}\Rightarrow-\frac{3\pi}{2}\le 3\theta\le-\frac{\pi}{2}$

Now,

$\sin^{-1}(3x-4x^3)$

$=\sin^{-1}(\sin 3\theta)$

$=\sin^{-1}(-\pi-3\theta)$

$=-\pi-3\theta\ \left[-\frac{3\pi}{2}\le 3\theta\left(2\Rightarrow-\frac{\pi}{2}\le-\pi-3\theta\le\frac{\pi}{2}\right)\right]$

$=-\pi-3\sin^{-1}x$

129 **(d)**

We have,

$\frac{\tan\frac{6\pi}{15}-\tan\frac{\pi}{15}}{1+\tan\frac{6\pi}{15}\tan\frac{\pi}{15}}=\tan\frac{\pi}{3}$

$\Rightarrow\tan\frac{6\pi}{15}-\tan\frac{\pi}{15}=\sqrt{3}+\sqrt{3}\tan\frac{6\pi}{15}\tan\frac{\pi}{15}$

$\Rightarrow\tan\frac{2\pi}{5}-\tan\frac{\pi}{15}-\sqrt{3}\tan\frac{2\pi}{5}\tan\frac{\pi}{15}=\sqrt{3}$

130 **(a)**

$\tan\left\{\cos^{-1}\left(-\frac{2}{7}\right)-\frac{\pi}{2}\right\}=\tan\left\{\pi-\cos^{-1}\left(\frac{2}{7}\right)-\frac{\pi}{2}\right\}$

$=\tan\left\{\frac{\pi}{2}-\cos^{-1}\left(\frac{2}{7}\right)\right\}$

$=\tan\left\{\sin^{-1}\frac{2}{7}\right\}$

$=\tan\left\{\tan^{-1}\left(\frac{2}{3\sqrt{5}}\right)\right\}=\frac{2}{3\sqrt{5}}$

131 **(a)**

$\sin\left[\sin^{-1}\left(\frac{1}{3}\right)+\sec^{-1}(3)\right]+\cos\left[\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}(2)\right]$

$=\sin\left[\sin^{-1}\left(\frac{1}{3}\right)+\cos^{-1}\left(\frac{1}{3}\right)\right]+\cos\left[\tan^{-1}\left(\frac{1}{2}\right)+\cot^{-1}\left(\frac{1}{2}\right)\right]$

$=\sin\frac{\pi}{2}+\cos\frac{\pi}{2}$

$\left[\because\sin^{-1}x+\cos^{-1}x=\frac{\pi}{2}\text{ and }\tan^{-1}x+\cot^{-1}x=\frac{\pi}{2}\right]$

$=1$

132 **(a)**

Let $\tan^{-1}x=\theta$. Then, $x=\tan\theta$

Also,

$-\frac{1}{\sqrt{3}}<x<\frac{1}{\sqrt{3}}$

$\Rightarrow\frac{1}{\sqrt{3}}<\tan\theta<\frac{1}{\sqrt{3}}\Rightarrow-\frac{\pi}{6}<\theta<\frac{\pi}{6}\Rightarrow-\frac{\pi}{2}<3\theta<\frac{\pi}{2}$

Now,

$\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)=\tan^{-1}(\tan 3\theta)$

$\Rightarrow\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)=3\theta\ \left[\because-\frac{\pi}{2}<3\theta<\frac{\pi}{2}\right]$

$\Rightarrow \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right) = 3\tan^{-1}x$

133 **(b)**

Let $\cos^{-1}\left(\frac{4}{5}\right) = \theta$. Then, $\cos\theta = \frac{4}{5}$

$\therefore \sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right) = \sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-\frac{4}{5}}{2}}$

$= \frac{1}{\sqrt{10}}$

134 **(a)**

$\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$

$\Rightarrow \frac{3x+2x}{1-6x^2} = \frac{\pi}{4}$

$\Rightarrow 5x = 1-6x^2$

$\Rightarrow 6x^2+5x-1 = 0$

$\Rightarrow x = -1, \frac{1}{6}$

But when $x = -1$,

$\tan^{-1}2x = \tan^{-1}(-2) < 0$

And $\tan^{-1}3x = \tan^{-1}(-3) < 0$

This value will not satisfy the given equation

Hence, $x = \frac{1}{6}$

135 **(c)**

$\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} = \sin^{-1}\frac{4}{5} + \tan^{-1}\frac{2\left(\frac{1}{3}\right)}{1-\left(\frac{1}{2}\right)^2}$

$= \sin^{-1}\frac{4}{5} + \tan^{-1}\frac{3}{4} = \sin^{-1}\frac{4}{5} + \cos^{-1}\frac{4}{5} = \frac{\pi}{2}$

$\left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$

136 **(a)**

Given equation is

$2\cos^{-1}x + \sin^{-1}x = \frac{11\pi}{6}$

$\Rightarrow \cos^{-1}x + (\cos^{-1}x + \sin^{-1}x) = \frac{11\pi}{6}$

$\Rightarrow \cos^{-1}x + \frac{\pi}{2} = \frac{11\pi}{6}$

$\Rightarrow \cos^{-1}x = \frac{4\pi}{3}$

Which is not possible as $\cos^{-1}x \in [0,\pi]$.

137 **(c)**

$\cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\cos\frac{5\pi}{3}\right)$

$= \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left[\sin\left(\frac{\pi}{2} - \frac{5\pi}{3}\right)\right]$

$= \frac{5\pi}{3} + \frac{\pi}{2} - \frac{5\pi}{3} = \frac{\pi}{2}$

Alternate

Since, $\cos^{-1}x + \sin^{-1}x = \frac{\pi}{2}$

$\therefore \cos^{-1}\left(\cos\frac{5\pi}{3}\right) + \sin^{-1}\left(\sin\frac{5\pi}{3}\right) = \frac{\pi}{2}$

138 **(d)**

$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} - \frac{\pi}{6} = \frac{\pi}{6} = 30°$

139 **(c)**

We have,

$\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x$

$\Rightarrow \sin\{\sin^{-1}x + \sin^{-1}(1-x)\} = \sin(\cos^{-1}x)$

$\Rightarrow x\sqrt{1-(1-x)^2} + \sqrt{1-x^2}(1-x) = \sqrt{1-x^2}$

$\Rightarrow x\sqrt{1-(1-x)^2} = x\sqrt{1-x^2}$

$\Rightarrow x = 0$ or, $2x - x^2 = 1 - x^2 \Rightarrow x = 0$ or $x = \frac{1}{2}$

141 **(d)**

Given, $5\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 7\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$-4\tan^{-1}\left(\frac{2x}{1-x^2}\right) - \tan^{-1}x = 5\pi$

$\Rightarrow \quad 5(2\tan^{-1}x) + 7(2\tan^{-1}x) - 4(2\tan^{-1}x)$

$- \tan^{-1}x = 5\pi$

$\Rightarrow \quad 15\tan^{-1}x = 5\pi$

$\Rightarrow \tan^{-1}x = \frac{\pi}{3}$

$\therefore \quad x = \sqrt{3}$

142 **(c)**

$\because -\frac{\pi}{2} \le \sin^{-1}x \le \frac{\pi}{2}, -\frac{\pi}{2} \le \sin^{-1}y \le \frac{\pi}{2}$

And $-\frac{\pi}{2} \le \sin^{-1}z \le \frac{\pi}{2}$

Given that, $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \frac{3\pi}{2}$

Which is possible only when

$\sin^{-1}x = \sin^{-1}y = \sin^{-1}z = \frac{\pi}{2}$

Or $x = y = z = 1$

Put $p = q = 1$

Then $f(2) = f(1)f(1) = 2 \cdot 2 = 4$

And put $p = 1, q = 2$

Then, $f(3) = f(1)f(2) = 2 \cdot 2^2 = 8$

$\therefore x^{f(1)} + y^{f(2)} + z^{f(3)} - \frac{x+y+z}{x^{f(1)} + y^{f(2)} + z^{f(3)}}$

$= 1 + 1 + 1 - \frac{3}{1+1+1}$

$= 3 - 1 = 2$

143 **(a)**

Given, $\tan^{-1}\left(\frac{1}{\sqrt{\cos\alpha}}\right) - \tan^{-1}(\sqrt{\cos\alpha}) = x$

$\Rightarrow \quad \tan^{-1}\left(\frac{\frac{1}{\sqrt{\cos\alpha}} - \sqrt{\cos\alpha}}{1 + \frac{1}{\sqrt{\cos\alpha}}.\sqrt{\cos\alpha}}\right) = x$

$\Rightarrow \quad \frac{1-\cos\alpha}{2\sqrt{\cos\alpha}} = \tan x$

$\Rightarrow \quad \frac{2\sqrt{\cos\alpha}}{1-\cos\alpha} = \cot x$

$$\Rightarrow \ \text{cosec}\, x = \frac{1+\cos\alpha}{1-\cos\alpha}$$

$$\Rightarrow \sin x = \frac{1-\cos\alpha}{1+\cos\alpha}$$

$$\Rightarrow \sin x = \frac{2\sin^2\left(\frac{\alpha}{2}\right)}{2\cos^2\left(\frac{\alpha}{2}\right)} = \tan^2\left(\frac{\alpha}{2}\right)$$

144 **(b)**

$$\cot^{-1} 9 + \text{cosec}^{-1}\frac{\sqrt{41}}{4} = \tan^{-1}\frac{1}{9} + \tan^{-1}\frac{1}{\sqrt{\frac{41}{16}-1}}$$

$$\left[\because \text{cosec}^{-1} x = \tan^{-1}\frac{1}{\sqrt{x^2-1}}\right]$$

$$= \tan^{-1}\frac{1}{9} + \tan^{-1}\frac{4}{5}$$

$$= \tan^{-1}\left(\frac{\frac{1}{9}+\frac{4}{5}}{1-\frac{1}{9}\cdot\frac{4}{5}}\right)$$

$$= \tan^{-1}\left(\frac{41}{41}\right) = \frac{\pi}{4}$$

145 **(a)**

We have, $\sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{m^4+m^2+2}\right)$

$$= \sum_{m=1}^{n} \tan^{-1}\left(\frac{2m}{1+(m^2+m+1)(m^2-m+1)}\right)$$

$$= \sum_{m=1}^{n} \tan^{-1}\left(\frac{(m^2+m+1)-(m^2-m+1)}{1+(m^2+m+1)(m^2-m+1)}\right)$$

$$= \sum_{m=1}^{n} [\tan^{-1}(m^2+m+1) - \tan^{-1}(m^2-m+1)]$$

$= (\tan^{-1} 3 - \tan^{-1} 1) + (\tan^{-1} 7 - \tan^{-1} 3) +$
$(\tan^{-1} 13 - \tan^{-1} 7) + \ldots +$
$[\tan^{-1}(n^2+n+1) - \tan^{-1}(n^2-n+1)]$

$$= \tan^{-1}\frac{n^2+n+1-1}{1+(n^2+n+1)\cdot 1}$$

$$= \tan^{-1}\left(\frac{n^2+n}{2+n^2+n}\right)$$

146 **(b)**

$$\therefore \cos^{-1}\frac{3}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1} x$$

$$\Rightarrow \sin^{-1}\frac{4}{5} - \sin^{-1}\frac{4}{5} = \cos^{-1} x$$

$\Rightarrow \cos^{-1} x = 0 \Rightarrow x = \cos 0 = 1$

$\therefore x = 1$

147 **(a)**

Given, $\cot(\cos^{-1} x) = \sec\left(\tan^{-1}\frac{a}{\sqrt{b^2-a^2}}\right)$

$$\therefore \ \cot\left(\cot^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right)$$

$$= \sec\left(\sec^{-1}\frac{b}{\sqrt{b^2-a^2}}\right)$$

$$\Rightarrow \quad \frac{x}{\sqrt{1-x^2}} = \frac{b}{\sqrt{b^2-a^2}}$$

$\Rightarrow \quad x^2(b^2-a^2) = b^2 - b^2x^2$

$\Rightarrow \ x^2(2b^2-a^2) = b^2$

$$\Rightarrow \ x = \frac{b}{\sqrt{2b^2-a^2}}$$

148 **(b)**

Given, $\sin^{-1} x - \cos^{-1} x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\Rightarrow \quad \sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$ ...(i)

But $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$\sin^{-1} x = \frac{\pi}{3}$ and $\cos^{-1} x = \frac{\pi}{6}$

$\Rightarrow \ x = \frac{\sqrt{3}}{2}$ is the unique solution.

149 **(d)**

We have, $\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x$

$= \frac{\pi}{2} - \tan^{-1} x = \cot^{-1} x$

Since, $0 \le x \le 1$, therefore $\frac{\pi}{4} \le \theta \le \frac{\pi}{2}$

150 **(a)**

$$\therefore \cot^{-1} x + \sin^{-1}\frac{1}{\sqrt{5}} = \frac{\pi}{4}$$

$$\Rightarrow \tan^{-1}\frac{1}{x} + \tan^{-1}\frac{1}{2} = \tan^{-1} 1$$

$$\Rightarrow \tan^{-1}\frac{1}{x} = \tan^{-1} 1 - \tan^{-1}\frac{1}{2}$$

$$\Rightarrow \tan^{-1}\frac{1}{x} = \tan^{-1}\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right)$$

$$\Rightarrow \tan^{-1}\frac{1}{x} = \tan^{-1}\frac{1}{3}$$

$\Rightarrow x = 3$

151 **(b)**

$$\sin\left(\frac{1}{2}\cos^{-1}\frac{4}{5}\right)$$

Now, put $\frac{4}{5} = \cos 2\theta$

$$\therefore \ \sin\left(\frac{1}{2} \times 2\theta\right)$$

$$= \sin\theta = \sqrt{\frac{1-\cos 2\theta}{2}}$$

$$= \sqrt{\frac{1-\frac{4}{5}}{2}}$$

$$= \sqrt{\frac{1}{5\times 2}}$$

$= \frac{1}{\sqrt{10}}$

152 **(b)**

Given, $\sin^{-1}\left(\frac{3}{x}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{x}\right)$

$\Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \cos^{-1}\left(\frac{4}{x}\right)$

$\Rightarrow \sin^{-1}\left(\frac{3}{x}\right) = \sin^{-1}\left(\frac{\sqrt{x^2-16}}{x}\right)$

$\Rightarrow \frac{3}{x} = \frac{\sqrt{x^2-16}}{x}$

$\Rightarrow x = \pm 5$

$\therefore x = 5$

[$\because -5$ not satisfies the given equation]

153 **(b)**

$\because 0 \le \cos^{-1} x \le \pi$

And $0 < \cot^{-1} x < \pi$

Given, $[\cot^{-1} x] + [\cot^{-1} x] = 0$

$\Rightarrow [\cot^{-1} x] = 0$ and $[\cos^{-1} x] = 0$

$\Rightarrow 0 < \cot^{-1} x < 1$ and $0 \le \cos^{-1} x < 1$

$\therefore x \in (\cot 1, \infty)$ and $x \in (\cos 1, 1)$

$\Rightarrow x \in (\cot 1, 1)$

154 **(b)**

$$3\sin^{-1}\frac{2x}{1+x^2} - 4\cos^{-1}\frac{1-x^2}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$$

On putting $x = \tan\theta$, we get

$$3\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) - 4\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right) + 2\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right) = \frac{\pi}{3}$$

$$\Rightarrow 3\sin^{-1}(\sin 2\theta) - 4\cos^{-1}(\cos 2\theta) + 2\tan^{-1}(\tan 2\theta) = \frac{\pi}{3}$$

$\Rightarrow 3(2\theta) - 4(2\theta) + 2(2\theta) = \frac{\pi}{3}$

$\Rightarrow 6\theta - 8\theta + 4\theta = \frac{\pi}{3}$

$\Rightarrow \theta = \frac{\pi}{6} \Rightarrow \tan^{-1} x = \frac{\pi}{6}$

$\Rightarrow x = \tan\frac{\pi}{6} \Rightarrow x = \frac{1}{\sqrt{3}}$

155 **(b)**

Here, $T_n = \cot^{-1}\left(n^2 + \frac{3}{4}\right)$

$= \tan^{-1}\left(\frac{4}{4n^2+3}\right)$

$= \tan^{-1}\left(\frac{1}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right)$

$= \tan^{-1}\left[\frac{\left(n+\frac{1}{2}\right)-\left(n-\frac{1}{2}\right)}{1+\left(n+\frac{1}{2}\right)\left(n-\frac{1}{2}\right)}\right]$

$= \tan^{-1}\left(n+\frac{1}{2}\right) - \tan^{-1}\left(n-\frac{1}{2}\right)$

$\therefore S_\infty = T_\infty^{-1} - \tan^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow S_\infty = \cot^{-1}\left(\frac{1}{2}\right)$

$\Rightarrow S_\infty = \tan^{-1}(2)$

156 **(d)**

$2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left[\frac{2\left(\frac{1}{3}\right)}{1-\frac{1}{9}}\right] + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\left(\frac{1}{7}\right)$

$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{1}{7}}{1-\frac{3}{4}\times\frac{1}{7}}\right)$

$= \tan^{-1}\left(\frac{25}{25}\right) = \frac{\pi}{4}$

157 **(c)**

The given equation is satisfied only when $x = 1$, $y = -1, z = 1$

158 **(d)**

Let $\cot^{-1} x = \theta \Rightarrow x = \cot\theta$

Now, $\operatorname{cosec}\theta = \sqrt{1+\cot^2\theta} = \sqrt{1+x^2}$

$\Rightarrow \sin\theta = \frac{1}{\operatorname{cosec}\theta} = \frac{1}{\sqrt{1+x^2}}$

$\Rightarrow \theta = \sin^{-1}\frac{1}{\sqrt{1+x^2}}$

$\therefore \sin(\cot^{-1} x) = \sin\left(\sin^{-1}\frac{1}{\sqrt{1+x^2}}\right)$

$= \frac{1}{\sqrt{1+x^2}} = (1+x^2)^{-1/2}$

159 **(c)**

$\therefore \tan^{-1} x - \tan^{-1} y = 0 \Rightarrow x = y$

Also, $\cos^{-1} x + \cos^{-1} y = \frac{\pi}{2} \Rightarrow 2\cos^{-1} x = \frac{\pi}{2}$

$\Rightarrow \cos^{-1} x = \frac{\pi}{4} \Rightarrow x = \frac{1}{\sqrt{2}} \Rightarrow x^2 = \frac{1}{2}$

Hence, $x^2 + xy + y^2 = 3x^2 = \frac{3}{2}$

160 **(a)**

Let $\tan^{-1} x = \theta$. Then, $x = \tan\theta$

Also, $-1 < x < 1 \Rightarrow -1 < \tan\theta < 1 \Rightarrow -\frac{\pi}{4} < \theta < \frac{\pi}{4}$

Now,

$\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}(\tan 2\theta)$

$= 2\theta \quad \left[\because -\frac{\pi}{4} < \theta < \frac{\pi}{4} \Rightarrow -\frac{\pi}{2} < 2\theta < \frac{\pi}{2}\right]$

$= 2\tan^{-1} x$

161 **(c)**

Given that, $\theta = \tan^{-1} a$ and $\phi = \tan^{-1} b$

And $ab = -1$

$\therefore \tan\theta \tan\phi = ab = -1$

$\Rightarrow \tan\theta = -\cot\phi$

$\Rightarrow \tan\theta = \tan\left(\frac{\pi}{2} + \phi\right)$

$\Rightarrow \theta - \phi = \frac{\pi}{2}$

162 **(b)**

Let $\cot^{-1}\frac{1}{2} = \phi \Rightarrow \frac{1}{2} = \cot\phi$

$\Rightarrow \sin\phi = \frac{1}{\sqrt{1+\cot^2\phi}} = \frac{2}{\sqrt{5}}$

Let $\cos^{-1} x = \theta \Rightarrow \sec\theta = \frac{1}{x}$

$\Rightarrow \tan\theta = \sqrt{\sec^2\theta - 1}$

$\Rightarrow \tan\theta = \sqrt{\frac{1}{x^2} - 1}$

$\Rightarrow \tan\theta = \frac{\sqrt{1-x^2}}{x}$

Now, $\tan(\cos^{-1} x) = \sin\left(\cot^{-1}\frac{1}{2}\right)$

$\Rightarrow \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{5}}\right)$

$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$

$\Rightarrow \sqrt{(1-x^2)5} = 2x$

On squaring both sides, we get

$(1-x^2)5 = 4x^2$

$\Rightarrow 9x^2 = 5$

$\Rightarrow x = \pm\frac{\sqrt{5}}{3}$

163 **(b)**

We have, $\sin^{-1} x + \sin^{-1} y = \pi - \sin^{-1} z$

or $x\sqrt{1-y^2} + y\sqrt{1-x^2} = z$

or $x^2(1-y^2) = z^2 + y^2(1-x^2) - 2yz\sqrt{(1-x^2)}$

or $(x^2 - z^2 - y^2)^2 = 4y^2z^2(1-x^2)$

or $x^4 + y^4 + z^4 - 2x^2z^2 + 2y^2z^2 - 2x^2y^2 + 4x^2y^2z^2 - 4y^2z^2 = 0$

or $x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

$\therefore k = 2$

164 **(c)**

Let $x = \tan\theta \quad \Rightarrow \quad \theta = \tan^{-1} x$

$\therefore \tan^{-1}\left(\frac{3x - x^3}{1-3x^2}\right) - \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

$= 3\tan^{-1} x - 2\tan^{-1} x$

$= \tan^{-1} x$

165 **(d)**

Let $\alpha = \cos^{-1}\sqrt{P}, \beta = \cos^{-1}\sqrt{1-P}$

And $\gamma = \cos^{-1}\sqrt{1-q}$

$\Rightarrow \cos\alpha = \sqrt{p},\ \cos\beta = \sqrt{1-p}$

And $\cos\gamma = \sqrt{1-q}$

Therefore, $\sin\alpha = \sqrt{1-p}, \sin\beta = \sqrt{p}$ and $\sin\gamma = \sqrt{q}$

The given equation may be written as

$\alpha + \beta + \gamma = \frac{3\pi}{4}$

$\Rightarrow \alpha + \beta = \frac{3\pi}{4} - \gamma$

$\Rightarrow \cos(\alpha+\beta) = \cos\left(\frac{3\pi}{4} - \gamma\right)$

$\Rightarrow \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \cos\left\{\pi - \left(\frac{\pi}{4} + \gamma\right)\right\} = -\cos\left(\frac{\pi}{4} + \gamma\right)$

$\Rightarrow \sqrt{p}\sqrt{1-p} - \sqrt{1-p}\sqrt{p}$

$= -\left(\frac{1}{\sqrt{2}}\sqrt{1-q} - \frac{1}{\sqrt{2}}\sqrt{q}\right)$

$\Rightarrow 0 = \sqrt{1-q} - \sqrt{q} \Rightarrow 1 - q = q$

$\Rightarrow q = \frac{1}{2}$

166 **(b)**

Let α, β are the roots of given equation $6x^2 - 5x + 1 = 0$

$\Rightarrow \quad \alpha + \beta = \frac{5}{6}$ and $\alpha\beta = \frac{1}{6}$

$\therefore \tan^{-1}\alpha + \tan^{-1}\beta = \tan^{-1}\left(\frac{\alpha+\beta}{1-\alpha\beta}\right)$

$= \tan^{-1}\left(\frac{\frac{5}{6}}{1-\frac{1}{6}}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

167 **(a)**

Since, $\alpha = \sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{1}{3}\right)$

$= \sin^{-1}\left(\frac{4}{5}\sqrt{1-\frac{1}{9}} + \frac{1}{3}\sqrt{1-\frac{16}{25}}\right)$

$\Rightarrow \alpha = \sin^{-1}\left(\frac{8\sqrt{2}}{15} + \frac{3}{15}\right) = \sin^{-1}\left(\frac{8\sqrt{2}+3}{15}\right)$

Since, $\frac{8\sqrt{2}+3}{15} < 1$

$\therefore \alpha < \frac{\pi}{2}$

Now, $\beta = \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{1}{3}\right)$

$\Rightarrow \beta = \frac{\pi}{2} - \sin^{-1}\left(\frac{4}{5}\right) + \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{3}\right)$

$= \pi - \left(\sin^{-1}\frac{4}{5} + \sin^{-1}\frac{1}{3}\right)$

$= \pi - \alpha$

$\Rightarrow \beta > \alpha \quad \left(\because \alpha < \frac{\pi}{2}\right)$

168 **(c)**

$\because [\sin^{-1} x] > [\cos^{-1} x]$

$\Rightarrow x > 0$

Here, $[\cos^{-1} x] = \begin{cases} 0, & x \in (\cos 1, 1) \\ 1, & x \in (0, \cos 1) \end{cases}$

and, $[\sin^{-1} x] = \begin{cases} 0, & x \in (0, \sin 1) \\ 1, & x \in (\sin 1, 1) \end{cases}$

$\therefore x \in [\sin 1, 1)$

$\because \left[\frac{x}{2}\right] = 1$

Or we say that $x \in [\sin 1, 1]$

169 **(c)**

We have,

$\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3$

$= \tan^{-1} 1 + \pi + \tan^{-1}\left(\frac{2+3}{1-2\times 3}\right)$

$= \tan^{-1} 1 + \pi + \tan^{-1}(-1) = \pi$

170 **(d)**

We have, $(\sin^{-1} x)^3 + (\cos^{-1} x)^3$

$= (\sin^{-1} + \cos^{-1} x)^3 - 3\sin^{-1} x \cos^{-1} x (\sin^{-1} x + \cos^{-1} x)$

$= \frac{\pi^3}{8} - 3(\sin^{-1} x \cos^{-1} x)\frac{\pi}{2}$

$= \frac{\pi^3}{8} - \frac{3\pi}{2}\sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right)$

$= \frac{\pi^3}{8} - \frac{3\pi}{2}\sin^{-1} x + \frac{3\pi}{2}(\sin^{-1} x)^2$

$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[(\sin^{-1} x)^2 - \frac{\pi}{2}\sin^{-1} x\right]$

$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[(\sin^{-1} x)^2 - \frac{\pi}{2}\sin^{-1} x + \frac{\pi^2}{16} - \frac{\pi^2}{16}\right]$

$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[\left(\sin^{-1} x - \frac{\pi}{4}\right)^2\right] - \frac{3\pi^3}{32}$

$= \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\sin^{-1} x - \frac{\pi}{4}\right)^2$

$\therefore$ The least value is $\frac{\pi^3}{32}$

Since, $\left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \le \left(\frac{3\pi}{4}\right)^2$

$\therefore$ The greatest value is $\frac{\pi^3}{32} + \frac{9\pi^2}{16} \times \frac{3\pi}{2} = \frac{7\pi^3}{8}$

171 **(d)**

Given, $\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{3}{4}\right) = \tan^{-1}\left(\frac{x}{3}\right)$

$\Rightarrow \tan^{-1}\left(\frac{\frac{1}{3}+\frac{3}{4}}{1-\frac{1}{3}\times\frac{3}{4}}\right) = \tan^{-1}\left(\frac{x}{3}\right)$

$\Rightarrow \frac{13}{9} = \frac{x}{3} \Rightarrow x = \frac{13}{3}$

173 **(b)**

$\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$

$= \tan^{-1}\frac{x}{y} - \tan^{-1}\left[\frac{1-\frac{y}{x}}{1+\frac{y}{x}}\right]$

$= \tan^{-1}\frac{x}{y} - \tan^{-1} 1 + \tan^{-1}\frac{y}{x}$

$= \tan^{-1}\frac{x}{y} + \cot^{-1}\frac{x}{y} - \tan^{-1} 1$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

174 **(c)**

Given, two angles of triangle are $\tan^{-1} 2$ and $\tan^{-1} 3$.

Let third angle be θ. Then,

$\tan^{-1} 2 + \tan^{-1} 3 + \theta = 180°$

$\Rightarrow \tan^{-1}\left(\frac{2+3}{1-2\times 3}\right) = 180° - \theta$

$\Rightarrow \frac{5}{-5} = \tan(180° - \theta) = -\tan\theta$

$\Rightarrow \tan\theta = 1 = \tan\frac{\pi}{4}$

$\Rightarrow \theta = \frac{\pi}{4}$

175 **(c)**

$8x^2 + 22x + 5 = 0 \Rightarrow x = -\frac{1}{4}, -\frac{5}{2}$

$\therefore -1 < -\frac{1}{4} < 1$ and $-\frac{5}{2} < -1$

$\therefore \sin^{-1}\left(-\frac{1}{4}\right)$ exists but $\sin^{-1}\left(-\frac{5}{2}\right)$ does not exist.

$\sec^{-1}\left(-\frac{5}{2}\right)$ exists but $\sec^{-1}\left(-\frac{1}{4}\right)$ does not exist.

$\tan^{-1}\left(-\frac{1}{4}\right)$ and $\tan^{-1}\left(-\frac{5}{2}\right)$ both exist.

176 **(b)**

We have, $\Sigma x_1 = \sin 2\beta, \Sigma x_1x_2 = \cos 2\beta, \Sigma x_1x_2x_3 = \cos\beta$ and $x_1x_2x_3x_4 = -\sin\beta$

$\therefore \tan^{-1} x_1 + \tan^{-1} x_2 + \tan^{-1} x_3 + \tan^{-1} x_4$

$= \tan^{-1}\left(\frac{\Sigma x_1 - \Sigma x_1x_2x_3}{1 - \Sigma x_1x_2 + x_1x_2x_3x_4}\right)$

$= \tan^{-1}\left(\frac{\sin 2\beta - \cos\beta}{1 - \cos 2\beta - \sin\beta}\right)$

$= \tan^{-1}\left(\frac{(2\sin\beta - 1)\cos\beta}{\sin\beta(2\sin\beta - 1)}\right)$

$= \tan^{-1}(\cot\beta)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{2} - \beta\right)\right) = \frac{\pi}{2} - \beta$

177 **(d)**

$\cos(2\cos^{-1}x+\sin^{-1}x)$
$=\cos[2(\cos^{-1}x+\sin^{-1}x)-\sin^{-1}x]$
$=\cos(\pi-\sin^{-1}x)=-\cos(\sin^{-1}x)$
$=-\cos\left[\sin^{-1}\left(-\frac{1}{5}\right)\right]\quad\left(\because x=\frac{1}{5}\right)$
$=-\cos\left(\cos^{-1}\frac{2\sqrt{6}}{5}\right)$
$=-\frac{2\sqrt{6}}{5}$

178 **(a)**

$\therefore \cot^{-1}x+\sin^{-1}\frac{1}{\sqrt{5}}=\frac{\pi}{4}$
$\Rightarrow \tan^{-1}\frac{1}{x}+\tan^{-1}\frac{1}{2}=\tan^{-1}1$
$\Rightarrow \tan^{-1}\frac{1}{x}=\tan^{-1}1-\tan^{-1}\frac{1}{2}$
$\Rightarrow \tan^{-1}\frac{1}{x}=\tan^{-1}\left(\frac{1-\frac{1}{2}}{1+\frac{1}{2}}\right)$
$\Rightarrow \tan^{-1}\frac{1}{x}=\tan^{-1}\frac{1}{3}$
$\Rightarrow x=3$

179 **(b)**

We have,
$\cos(2\tan^{-1}x)=\frac{1}{2}$
$\Rightarrow 2\tan^{-1}x=\frac{\pi}{3}\Rightarrow \tan^{-1}x=\frac{\pi}{6}\Rightarrow x=\tan\frac{\pi}{6}=\frac{1}{\sqrt{3}}$

180 **(c)**

Given that, $\sin^{-1}\frac{1}{3}+\sin^{-1}\frac{2}{3}=\sin^{1}x$
$\Rightarrow \sin^{-1}\left(\frac{1}{3}\sqrt{1-\frac{4}{9}}+\frac{2}{3}\sqrt{1-\frac{1}{9}}\right)=\sin^{-1}x$
$\Rightarrow \sin^{-1}\left(\frac{1}{3}.\frac{\sqrt{5}}{3}+\frac{2}{3}.\frac{\sqrt{8}}{3}\right)=\sin^{-1}x$
$\Rightarrow \sin^{-1}\left(\frac{\sqrt{5}+4\sqrt{2}}{9}\right)=\sin^{-1}x$
$\therefore x=\left(\frac{\sqrt{5}+4\sqrt{2}}{9}\right)$

181 **(c)**

Since, $\tan^{-1}x$ and $\cot^{-1}x$ exists for all $x\in R$ and $\cos^{-1}(2-x)$ exists, if $-1\le 2-x\le 1$
$\therefore \tan^{-1}x-\cot^{-1}x=\cos^{-1}(2-x)$
Is possible only if $1\le x\le 3$.
Thus the solution of given equation is [1, 3].

182 **(a)**

Since, $0\le \cos^{-1}\left(\frac{x^2}{2}+\sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\right)\le\frac{\pi}{2}$
Because $\cos^{-1}x$ is in first quadrant when x is positive
And $\cos^{-1}\frac{x}{2}-\cos^{-1}x\ge 0$
So, $\cos^{-1}\frac{x}{2}\ge\cos^{-1}x$
Also, $\left|\frac{x}{2}\right|\le 1, |x|\le 1\Rightarrow |x|\le 1$

183 **(b)**

We have,
$\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{8}{31}\right)$
$\Rightarrow \tan^{-1}\left\{\frac{2x}{1-(x^2-1)}\right\}=\tan^{-1}\left(\frac{8}{31}\right)$
$\Rightarrow \frac{2x}{2-x^2}=\frac{8}{31}$
$\Rightarrow 8x^2+62x-16=0\Rightarrow(4x-1)(x+8)=0$
$\Rightarrow x=\frac{1}{4},-8$

184 **(b)**

$3\sin^{-1}\frac{2x}{1+x^2}-4\cos^{-1}\frac{1-x^2}{1+x^2}+2\tan^{-1}\frac{2x}{1-x^2}=\frac{\pi}{3}$
On putting $x=\tan\theta$, we get
$3\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)-4\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)+2\tan^{-1}\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)=\frac{\pi}{3}$
$\Rightarrow 3\sin^{-1}(\sin 2\theta)-4\cos^{-1}(\cos 2\theta)+2\tan^{-1}(\tan 2\theta)=\frac{\pi}{3}$
$\Rightarrow 3(2\theta)-4(2\theta)+2(2\theta)=\frac{\pi}{3}$
$\Rightarrow 6\theta-8\theta+4\theta=\frac{\pi}{3}$
$\Rightarrow \theta=\frac{\pi}{6}\Rightarrow \tan^{-1}x=\frac{\pi}{6}$
$\Rightarrow x=\tan\frac{\pi}{6}\Rightarrow x=\frac{1}{\sqrt{3}}$

185 **(b)**

Given that, $x^2+y^2+z^2=r^2$
Now, $\tan^{-1}\left(\frac{xy}{zr}\right)+\tan^{-1}\left(\frac{yz}{xe}\right)+\tan^{-1}\left(\frac{xz}{yr}\right)$
$=\tan^{-1}\left[\frac{\frac{xy}{zr}+\frac{yz}{xr}+\frac{xz}{yr}-\frac{xyz}{r^3}}{1-\left(\frac{x^2+y^2+z^2}{r^2}\right)}\right]$
$=\tan^{-1}\left[\frac{\frac{xy}{zr}+\frac{yz}{xr}+\frac{xz}{yr}-\frac{xyz}{r^3}}{1-\frac{r^2}{r^2}}\right]$
$=\tan^{-2}\infty=\frac{\pi}{2}$

186 **(d)**

We have, $(\sin^{-1}x)^3+(\cos^{-1}x)^3$

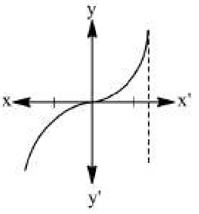
$= (\sin^{-1} + \cos^{-1} x)^3$
$- 3\sin^{-1} x \cos^{-1} x (\sin^{-1} x + \cos^{-1} x)$
$= \frac{\pi^3}{8} - 3(\sin^{-1} x \cos^{-1} x)\frac{\pi}{2}$
$= \frac{\pi^3}{8} - \frac{3\pi}{2}\sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right)$
$= \frac{\pi^3}{8} - \frac{3\pi}{2}\sin^{-1} x + \frac{3\pi}{2}(\sin^{-1} x)^2$
$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[(\sin^{-1} x)^2 - \frac{\pi}{2}\sin^{-1} x\right]$
$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[(\sin^{-1} x)^2 - \frac{\pi}{2}\sin^{-1} x + \frac{\pi^2}{16} - \frac{\pi^2}{16}\right]$
$= \frac{\pi^3}{8} + \frac{3\pi}{2}\left[\left(\sin^{-1} x - \frac{\pi}{4}\right)^2\right] - \frac{3\pi^3}{32}$
$= \frac{\pi^3}{32} + \frac{3\pi}{2}\left(\sin^{-1} x - \frac{\pi}{4}\right)^2$

$\therefore$ The least value is $\frac{\pi^3}{32}$

Since, $\left(\sin^{-1} x - \frac{\pi}{4}\right)^2 \leq \left(\frac{3\pi}{4}\right)^2$

$\therefore$ The greatest value is $\frac{\pi^3}{32} + \frac{9\pi^2}{16} \times \frac{3\pi}{2} = \frac{7\pi^3}{8}$

187 **(d)**



Hence, the line $x = 1$ is a tangent to the function.

188 **(c)**

Let $\sin^{-1} x = \theta$. Then, $x = \sin\theta$ and $\sqrt{1-x^2} = \cos\theta$

Now,

$-1 \leq x \leq -\frac{1}{\sqrt{2}} \Rightarrow -1 \leq \sin\theta \leq -\frac{1}{\sqrt{2}} \Rightarrow -\frac{\pi}{2} \leq \theta \leq -\frac{\pi}{4}$

$\therefore \sin^{-1}\left(2x\sqrt{1-x^2}\right) = \sin^{-1}(\sin 2\theta)$
$= \sin^{-1}(-\sin(\pi + 2\theta))$
$= \sin^{-1}(\sin(-\pi - 2\theta))$
$= -\pi - 2\theta \left[\because -\frac{\pi}{2} \leq \theta \leq -\frac{\pi}{4} \Rightarrow -\frac{\pi}{2} \leq -\pi - 2\theta \leq 0\right]$
$= -\pi - 2\sin^{-1} x$

189 **(a)**

$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$

Let $s^2 = \frac{a+b+c}{abc}$

Hence, $\theta = \tan^{-1}\sqrt{a^2s^2} + \tan^{-1}\sqrt{b^2s^2} + \tan^{-1}\sqrt{c^2s^2}$
$= \tan^{-1}(as) + \tan^{-1}(bs) + \tan^{-1}(cs)$
$= \tan^{-1}\left[\frac{as + bs + cs - abcs^3}{1 - abs^2 - acs^2 - bcs^2}\right]$

Hence, $\tan\theta = \left[\frac{s[a+b+c) - abcs^2]}{1-(ab+bc+ca)s^2}\right]$
$= \left[\frac{s[(a+b+c) - (a+b+c)]}{1 - s^2(ab+bc+ca)}\right] = 0$

190 **(a)**

We have,

$\theta \in [4\pi, 5\pi] \Rightarrow -4\pi + \theta \in [0, \pi]$

Also,

$\cos(-4\pi + \theta) = \cos(4\pi - \theta) = \cos\theta$

$\therefore \cos^{-1}(\cos\theta) = \cos^{-1}\{\cos(-4\pi + \theta)\}$
$= -4\pi + \theta$

191 **(c)**

Given, $\sin^{-1} x = 2\sin^{-1} a$

Since, $-\frac{\pi}{2} \leq \sin^{-1} x \leq \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{2} \leq 2\sin^{-1} a \leq \frac{\pi}{2}$

$\Rightarrow -\frac{\pi}{4} \leq \sin^{-1} a \leq \frac{\pi}{4}$

$\Rightarrow \sin\left(-\frac{\pi}{4}\right) \leq a \leq \sin\frac{\pi}{4}$

$\Rightarrow -\frac{1}{\sqrt{2}} \leq a \leq \frac{1}{\sqrt{2}}$

$\therefore |a| \leq \frac{1}{\sqrt{2}}$

192 **(b)**

We have,

$\sin^{-1} x + \sin^{-1} 2x = \frac{\pi}{3}$

$\Rightarrow \sin^{-1} 2x = \frac{\pi}{3} - \sin^{-1} x$

$\Rightarrow 2x = \sin\left(\frac{\pi}{3} - \sin^{-1} x\right)$

$\Rightarrow 2x = \frac{\sqrt{3}}{2}\cos(\sin^{-1} x) - \frac{1}{2}\sin(\sin^{-1} x)$

$\Rightarrow 2x = \frac{\sqrt{3}}{2} \times \sqrt{1-x^2} - \frac{x}{2}$

$\Rightarrow \frac{5x}{2} = \frac{\sqrt{3}}{2}\sqrt{1-x^2}$

$\Rightarrow 25x^2 = 3 - 3x^2$

$\Rightarrow x = \pm\frac{1}{2}\sqrt{\frac{3}{7}} \Rightarrow x = \frac{1}{2}\sqrt{\frac{3}{7}}$ $[\because \text{RHS} > 0 \ \therefore x > 0]$

193 **(c)**

Since, $2\sin^{-1} = \sin^{-1}(2x\sqrt{1-x^2})$

Range of right hand side is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow -\frac{\pi}{2} \le 2\sin^{-1}x \le \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{4} \le \sin^{-1}x \le \frac{\pi}{4}$

$\Rightarrow x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

194 **(c)**

$\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$

$\Rightarrow \cos^{-1}\left(xy - \sqrt{1-y^2}\sqrt{1-x^2}\right) = \pi - \cos^{-1}z$

$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = \cos(\pi - \cos^{-1}z)$

$\Rightarrow xy - \sqrt{1-x^2}\sqrt{1-y^2} = -z$

$\Rightarrow xy + z = \sqrt{1-x^2}\sqrt{1-y^2}$

On squaring both sides, we get

$x^2y^2 + z^2 + 2xyz - 1 - x^2 - y^2 + x^2y^2$

$\Rightarrow x^2 + y^2 + z^2 = 1 - 2xyz$

195 **(b)**

$\cot\left(\text{cosec}^{-1}\frac{5}{3} + \tan^{-1}\frac{2}{3}\right)$

$= \cot\left(\tan^{-1}\frac{3}{4} + \tan^{-1}\frac{2}{3}\right)$

$= \cot\tan^{-1}\left[\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{1}{2}}\right]$

$= \cot\left[\tan^{-1}\left(\frac{17}{6}\right)\right]$

$= \frac{6}{17}$

196 **(b)**

We have, $\sin^{-1}x + \sin^{-1}y = \pi - \sin^{-1}z$

or $x\sqrt{1-y^2} + y\sqrt{1-x^2} = z$

or $x^2(1-y^2) = z^2 + y^2(1-x^2) - 2yz\sqrt{(1-x^2)}$

or $(x^2 - z^2 - y^2)^2 = 4y^2z^2(1-x^2)$

or $x^4 + y^4 + z^4 - 2x^2z^2 + 2y^2z^2 - 2x^2y^2 + 4x^2y^2z^2 - 4y^2z^2 = 0$

or $x^4 + y^4 + z^4 + 4x^2y^2z^2 = 2(x^2y^2 + y^2z^2 + z^2x^2)$

$\therefore k = 2$

197 **(b)**

Given, $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$

$\Rightarrow \tan^{-1}\left(\frac{x+y+z-xyz}{1-xy-yz-zx}\right) = \pi$

$\Rightarrow \frac{x+y+z-xyz}{1-xy-yz-zx} = 0$

$\Rightarrow x + y + z = xyz$

198 **(b)**

Since, 1 radian $= \frac{7\pi}{22}$

$\therefore$ 12 radian $= \frac{7\pi}{22} \times 12 = \frac{42\pi}{11} = 4\pi - \frac{2\pi}{11}$

And 14 radian $= \frac{7\pi}{22} \times 14 = \frac{49\pi}{11}$

$= 4\pi + \frac{5\pi}{11}$

$\therefore \cos^{-1}(\cos 12) - \sin^{-1}(\sin 14)$

$= \cos^{-1}\cos\left(4\pi - \frac{2\pi}{11}\right) - \sin^{-1}\left[\sin\left(4\pi + \frac{5\pi}{11}\right)\right]$

$= \cos^{-1}\cos\left(\frac{2\pi}{11}\right) - \sin^{-1}\left(\sin\frac{5\pi}{11}\right)$

$= 4\pi - 12 - (14 - 4\pi) = 8\pi - 26$

199 **(b)**

Let $\sin^{-1}x = \theta$. Then, $x = \sin\theta$

Also,

$\frac{1}{2} \le x \le 1 \Rightarrow \frac{1}{2} \le \sin\theta \le 1 \Rightarrow \frac{\pi}{6} \le \theta \le \frac{\pi}{2} \Rightarrow \frac{\pi}{2} \le 3\theta \le \frac{3\pi}{2}$

Now,

$\sin^{-1}(3x - 4x^3)$

$= \sin^{-1}(\sin 3\theta)$

$= \sin^{-1}(\sin(\pi - 3\theta))$

$= \pi - 3\theta$ $\left[\because \frac{\pi}{2} \le 3\theta \le \frac{3\pi}{2} \Rightarrow -\frac{\pi}{2} \le \pi - 3\theta \le \frac{\pi}{2}\right]$

$= \pi - 3\sin^{-1}x$

200 **(c)**

$\cos(4095°) = \cos(45 \times 90° + 45°)$

$= -\sin 45°$

$= -\sin\frac{\pi}{4}$

$= \sin\left(-\frac{\pi}{4}\right)$

$\therefore \sin^{-1}\{\cos(4095°)\}$

$= \sin^{-1}\sin\left(-\frac{\pi}{4}\right)$

$= -\frac{\pi}{4}$

201 **(d)**

We have,

$\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \tan^{-1}\left\{\frac{1/2 + 2/9}{1 - 1/4 \times 2/9}\right\}$

$= \tan^{-1}\left(\frac{1}{2}\right)$

202 **(c)**

Since, $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$

$\therefore \sin^{-1}\alpha = \frac{\pi}{2}, \sin^{-1}\beta = \frac{\pi}{2}$ and $\sin^{-1}\gamma = \frac{\pi}{2}$

$\therefore \alpha = \beta = \gamma = 1$

Thus, $\alpha\beta + \alpha\gamma + \gamma\beta = 3$

203 **(d)**

We know that,

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$= \frac{\frac{x\sqrt{3}}{2k-x} - \frac{2x-k}{k\sqrt{3}}}{1 + \frac{x\sqrt{3}}{2k-x}\cdot\frac{2x-k}{k\sqrt{3}}} = \frac{1}{\sqrt{3}}$$

$\Rightarrow A - B = 30°$

204 **(c)**

Given that, $\angle A = \tan^{-1} 2, \angle B = \tan^{-1} 3$

We know that, $\angle A + \angle B + \angle C = \pi$

$\Rightarrow \tan^{-1} 2 + \tan^{-1} 3 + \angle C = \pi$

$\Rightarrow \tan^{-1}\left(\frac{2+3}{1 - 2 \times 3}\right) + \angle C = \pi$

$\Rightarrow \tan^{-1}(-1) + \angle C = \pi$

$\Rightarrow \frac{3\pi}{4} + \angle C = \pi$

$\Rightarrow \angle C = \frac{\pi}{4}$

205 **(a)**

We have,

$$(\tan^{-1} x)^2 + (\cot^{-1} x)^2 = \frac{5\pi^2}{8}$$

$$\Rightarrow (\tan^{-1} x + \cot^{-1} x)^2 - 2\tan^{-1} x\left(\frac{\pi}{2} - \tan^{-1} x\right) = \frac{5\pi^2}{8}$$

$$\Rightarrow \frac{\pi^2}{4} - 2 \times \frac{\pi}{2}\tan^{-1} x + 2(\tan^{-1} x)^2 = \frac{5\pi^2}{8}$$

$$\Rightarrow 2(\tan^{-1} x)^2 - \pi\tan^{-1} x - \frac{3\pi^2}{8} = 0$$

$$\Rightarrow \tan^{-1} x = -\frac{\pi}{6}, \frac{3\pi}{4} \Rightarrow \tan^{-1} x = -\frac{\pi}{4} \Rightarrow x = -1$$

206 **(d)**

$$4\tan^{-1}\frac{1}{5} = 2\left[2\tan^{-1}\frac{1}{5}\right]$$

$$= 2\tan^{-1}\frac{\frac{2}{5}}{1 - \frac{1}{25}} = 2\tan^{-1}\frac{5}{12}$$

$$= \tan^{-1}\frac{\frac{10}{12}}{1 - \frac{25}{144}}$$

$$= \tan^{-1}\frac{120}{119}$$

So, $4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239} = \tan^{-1}\frac{120}{119} - \tan^{-1}\frac{1}{239}$

$$= \tan^{-1}\frac{\frac{120}{119} - \frac{1}{239}}{1 + \frac{120}{119} \times \frac{1}{239}}$$

$$= \tan^{-1}\frac{(120 \times 239) - 119}{(119 \times 239) + 120}$$

$$= \tan^{-1}\frac{28561}{28561} = \tan^{-1} 1 = \frac{\pi}{4}$$

207 **(b)**

Given, $\sin^{-1} x - \cos^{-1} x = \frac{\pi}{6}$

$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1} x\right) - \cos^{-1} x = \frac{\pi}{6}$

$\Rightarrow \cos^{-1} x = \frac{\pi}{6}$

$\Rightarrow x = \frac{\sqrt{3}}{2}$

208 **(d)**

$\because f(x) = ax + b$

$\therefore f'(x) = a > 0$

$\Rightarrow f(x)$ is an increasing function.

$\therefore f(-1) = 0$ and $f(1) = 2$

Or $-a + b = 0$

and $a + b = 2$

then, $a = b = 1$

$\Rightarrow f(x) = x + 1$

Now, $\cot[\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18]$

$$= \cot\left\{\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{\frac{1}{7} + \frac{1}{8}}{1 - \frac{1}{7}\cdot\frac{1}{8}}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{15}{35}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{3}{11}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{\frac{3}{11} + \frac{1}{18}}{1 - \frac{3}{1\cdot 1}\cdot\frac{1}{18}}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{65}{195}\right)\right\}$$

$$= \cot\left\{\tan^{-1}\left(\frac{1}{3}\right)\right\}$$

$$= \cot(\cot^{-1} 3) = 3 = 1 + 2 = f(2)$$

209 **(d)**

$$\cos\left(\frac{33\pi}{5}\right) = \cos\left(6\pi + \frac{3\pi}{5}\right) = \cos\frac{3\pi}{5}$$

$$= \sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right) = \sin\left(-\frac{\pi}{10}\right)$$

$$= \sin^{-1}\sin\left(-\frac{\pi}{10}\right) = -\frac{\pi}{10}$$

210 **(c)**

Given equation is

$$\cos^{-1} x + \cos^{-1} 2x + \pi = 0$$

$\Rightarrow \quad \cos^{-1}x + \cos^{-1}2x = -\pi$

$\Rightarrow \cos^{-1}\left(x.2x - \sqrt{1-x^2}\sqrt{1-4x^2}\right) = -\pi$

$\Rightarrow \quad 2x^2 - \sqrt{1-x^2}\sqrt{1-4x^2} = -1$

$\Rightarrow \quad (1+2x^2) = \sqrt{1-x^2}\sqrt{1-4x^2}$

On squaring both sides, we get

$1 + 4x^2 + 4x^2 = (1-x^2)(1-4x^2)$

$\Rightarrow \quad 1 + 4x^4 + 4x^2 = 1 - 5x^2 + 4x^4$

$\Rightarrow \quad 9x^2 = 0$

$\Rightarrow \quad x = 0$

But $x = 0$ is not satisfied the given equation.

$\therefore$ The number of real solution is zero.

211 **(c)**

Let $\cos^{-1}\left(\frac{\sqrt{5}}{3}\right) = \alpha$. Then,

$\cos\alpha = \frac{\sqrt{5}}{3}$, where $0 < \alpha < \frac{\pi}{2}$

Now,

$\tan\frac{\alpha}{2} = \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$

$\Rightarrow \tan\frac{\alpha}{2} = \sqrt{\frac{1-\sqrt{5}/3}{1+\sqrt{5}/3}}$

$\Rightarrow \tan\frac{\alpha}{2} = \sqrt{\frac{3-\sqrt{5}}{3+\sqrt{5}}} = \sqrt{\frac{(3-\sqrt{5})^2}{9-5}} = \frac{1}{2}(3-\sqrt{5})$

$\therefore \tan\left\{\frac{1}{2}\cos^{-1}\left(\frac{\sqrt{5}}{3}\right)\right\} = \frac{3-\sqrt{5}}{2}$

212 **(c)**

$\sin\left[2\cos^{-1}\frac{\sqrt{5}}{3}\right] = \sin\left[\cos^{-1}\left\{2.\left(\frac{\sqrt{5}}{3}\right)^2 - 1\right\}\right]$

$[\because 2\cos^{-1}x = \cos^{-1}(2x^2-1)]$

$= \sin\left[\cos^{-1}\left(\frac{1}{9}\right)\right]$

$= \sin\left[\sin^{-1}\sqrt{1-\left(\frac{1}{9}\right)^2}\right]$

$[\because \cos^{-1}x = \sin^{-1}(\sqrt{1-x^2})]$

$= \frac{4\sqrt{5}}{9}$

213 **(c)**

Let $\tan^{-1}x = \theta$. Then, $x = \tan\theta$

Also,

$x < -\frac{1}{\sqrt{3}} \Rightarrow \tan\theta < -\frac{1}{\sqrt{3}} \Rightarrow -\frac{\pi}{2} < \theta < -\frac{\pi}{6}$

Now,

$\tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

$= \tan^{-1}(\tan 3\theta)$

$= \tan^{-1}(\tan(\pi+3\theta)) = \pi + 3\theta = \pi + 3\tan^{-1}x$

214 **(c)**

$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{3} + 2\cdot\frac{\pi}{6} = \frac{\pi}{3} + \frac{\pi}{3} = \frac{2\pi}{3}$

215 **(d)**

Given, $\sin\left[\sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}x\right] = 1$

$\therefore \quad \sin^{-1}\left(\frac{1}{5}\right) + \cos^{-1}x = \frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1}\left(\frac{1}{5}\right) = \sin^{-1}x$

$\Rightarrow \quad x = \frac{1}{5}$

216 **(c)**

Given that, $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$

$\because 0 \le \cos^{-1}x \le \pi$

Similarly, $0 \le \cos^{-1}y \le \pi$

And $0 \le \cos^{-1}z \le \pi$

Here, $\cos^{-1}x\cos^{-1}y = \cos^{-1}z = \pi$

$\Rightarrow x = y = z = \cos\pi = -1$

$\therefore xy + yz + zx$

$= (-1)(-1) + (-1)(-1) + (-1)(-1)$

$=1+1+1=3$

217 **(a)**

Let $\tan^{-1}x = \theta$. Then, $x = \tan\theta$

Also,

$0 \le x \le \infty \Rightarrow 0 \le \theta < \frac{\pi}{2} \Rightarrow 0 \le 2\theta < \pi$

Now,

$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \cos^{-1}(\cos 2\theta)$

$= 2\theta \quad \left[\because 0 \le \theta < \frac{\pi}{2} \Rightarrow 0 \le 2\theta < \pi\right]$

$= 2\tan^{-1}x$

218 **(d)**

$\cos[2\tan^{-1}(-7)] = \cos\left[\cos^{-1}\left(\frac{1-49}{1+49}\right)\right]$

$= \cos\left[\pi - \cos^{-1}\left(\frac{48}{50}\right)\right]$

$= -\cos\cos^{-1}\left(\frac{48}{50}\right)$

$= -\frac{24}{25}$

219 **(d)**

We have,

$\sin\left(4\tan^{-1}\frac{1}{3}\right)$

$= 2\sin\left(2\tan^{-1}\frac{1}{3}\right)\cos\left(2\tan^{-1}\frac{1}{3}\right)$

$= 2\sin\left(\tan^{-1}\frac{3}{4}\right)\cos\left(\tan^{-1}\frac{3}{4}\right)$

$= 2\sin\left(\sin^{-1}\frac{3}{5}\right)\cos\left(\cos^{-1}\frac{4}{5}\right) = 2\times\frac{3}{5}\times\frac{4}{5} = \frac{24}{25}$

And,

$\cos\left(2\tan^{-1}\frac{1}{7}\right) = \cos\left(\tan^{-1}\frac{7}{24}\right) = \cos\left(\cos^{-1}\frac{24}{25}\right)$

$= \frac{24}{25}$

Hence, the value of given expression is 0

220 **(c)**

Given that , $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$

$\because 0 \le \cos^{-1}x \le \pi$

Similarly, $0 \le \cos^{-1}y \le \pi$

And $0 \le \cos^{-1}z \le \pi$

Here, $\cos^{-1}x\cos^{-1}y = \cos^{-1}z = \pi$

$\Rightarrow x = y = z = \cos\pi = -1$

$\therefore xy + yz + zx$

$= (-1)(-1) + (-1)(-1) + (-1)(-1)$

=1+1+1=3

221 **(b)**

Given expression

$= \tan\left[\tan^{-1}\frac{a_2 - a_1}{1 + a_1a_2} + \tan^{-1}\frac{a_3 - a_2}{1 + a_2a_3} + \ldots + \tan^{-1}\frac{a_n - a_{n-1}}{1 + a_na_{n-1}}\right]$

$= \tan[\tan^{-1}a_2 - \tan^{-1}a_1 + \tan^{-1}a_3 - \tan^{-1}a_2 + \ldots + \tan^{-1}a_n - \tan^{-1}a_{n-1}]$

$= \tan[\tan^{-1}a_n - \tan^{-1}a_1] = \frac{a_n - a_1}{1 + a_1a_n}$

$= \frac{(n-1)d}{1 + a_1a_n}$

222 **(a)**

$\sin\left(2\sin^{-1}\sqrt{\frac{63}{65}}\right) = \sin\left(\sin^{-1}2\sqrt{\frac{63}{65}}\sqrt{1 - \frac{63}{65}}\right)$

$= \sin\left(\sin^{-1}\frac{2\sqrt{126}}{65}\right) = \frac{2\sqrt{126}}{65}$

223 **(b)**

$\sin^{-1}x - \cos^{-1}x = \frac{\pi}{6}$

$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1}x\right) - \cos^{-1}x = \frac{\pi}{6}$

$\Rightarrow 2\cos^{-1}x = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

$\Rightarrow \cos^{-1}x = \frac{\pi}{6} \Rightarrow x = \frac{\sqrt{3}}{2}$

224 **(c)**

We know that

$\sin^{-1}\left(\frac{2x}{1+x^2}\right) = 2\tan^{-1}x$ for all $x \in [-1, 1]$

And,

$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 2\tan^{-1}x$ for all $x \in [0, \infty)$

$\therefore \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = 4\tan^{-1}x$ for all $x \in [0, 1]$

225 **(c)**

$\tan^{-1}\left(\frac{c_1 - y}{c_1y + x}\right) + \tan^{-1}\left(\frac{c_2 - c_1}{1 + c_2c_1}\right) + \tan^{-1}\left(\frac{c_3 - c_2}{1 + c_3c_2}\right) + \ldots + \tan^{-1}\frac{1}{c_n}$

$= \tan^{-1}\left(\frac{\frac{x}{y} - \frac{1}{c_1}}{1 + \frac{x}{y}\cdot\frac{1}{c_1}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_1} - \frac{1}{c_2}}{1 + \frac{1}{c_1c_2}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_2} - \frac{1}{c_3}}{1 + \frac{1}{c_2c_3}}\right) + \ldots + \tan^{-1}\frac{1}{c_n}$

$= \tan^{-1}\frac{x}{y} - \tan^{-1}\frac{1}{c_1} + \tan^{-1}\frac{1}{c_1} - \tan^{-1}\frac{1}{c_2} + \tan^{-1}\frac{1}{c\_2} - \tan^{-1}\frac{1}{c_3} + \ldots + \tan^{-1}\frac{1}{c_{n-1}} - \tan^{-1}\frac{1}{c_n} + \tan^{-1}\frac{1}{c_n}$

$= \tan^{-1}\left(\frac{x}{y}\right)$

226 **(c)**

We have, $\tan^{-1}a + \tan^{-1}b = \sin^{-1}1 - \tan^{-1}c$

$\Rightarrow \tan^{-1}a + \tan^{-1}b + \tan^{-1}c = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\left\{\frac{a + b + c - abc}{1 - (ab + bc + ca)}\right\} = \frac{\pi}{2}$

$\Rightarrow ab + bc + ca = 1$

227 **(d)**

$\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}]$

$= \cos\left[\tan^{-1}\left\{\sin\left(\sin^{-1}\frac{1}{\sqrt{1 + x^2}}\right)\right\}\right]$

$= \cos\left[\tan^{-1}\frac{1}{\sqrt{1 + x^2}}\right]$

$= \cos\left[\cos^{-1}\sqrt{\frac{1 + x^2}{2 + x^2}}\right]$

$= \sqrt{\frac{1 + x^2}{2 + x^2}}$

228 **(b)**

$\because 0 \le \cos^{-1}x \le \pi$

And $0 < \cot^{-1}x < \pi$

Given, $[\cot^{-1} x] + [\cot^{-1} x] = 0$
$\Rightarrow [\cot^{-1} x] = 0$ and $[\cos^{-1} x] = 0$
$\Rightarrow 0 < \cot^{-1} x < 1$ and $0 \le \cos^{-1} x < 1$
$\therefore x \in (\cot 1, \infty)$ and $x \in (\cos 1, 1)$
$\Rightarrow x \in (\cot 1, 1)$

229 **(d)**
Given, $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = 3\pi$
And we know that $0 \le \cos^{-1} x \le \pi$
$\therefore$ We know
$\cos^{-1} x = \pi, \cos^{-1} y = \pi, \cos^{-1} z = \pi$
$\therefore \quad x = y = z = \cos \pi = -1$
$\therefore xy + yz + zx = (-1)(-1) + (-1)(-1) + (-1)(-1) = 3$

230 **(c)**
We have,
$\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$
$\Rightarrow \tan^{-1}(1+x) = \frac{\pi}{2} - \tan^{-1}(1-x)$
$\Rightarrow \tan^{-1}(1+x) = \cot^{-1}(1-x)$
$\Rightarrow \tan^{-1}(1+x) = \tan^{-1}\left(\frac{1}{1-x}\right)$
$\Rightarrow 1 + x = \frac{1}{1-x} \Rightarrow 1 - x^2 = 1 \Rightarrow x = 0$

232 **(a)**
Given equation is
$2\cos^{-1} x + \sin^{-1} x = \frac{11\pi}{6}$
$\Rightarrow \cos^{-1} x + (\cos^{-1} x + \sin^{-1} x) = \frac{11\pi}{6}$
$\Rightarrow \cos^{-1} x + \frac{\pi}{2} = \frac{11\pi}{6}$
$\Rightarrow \cos^{-1} x = \frac{4\pi}{3}$
Which is not possible as $\cos^{-1} x \in [0, \pi]$.

233 **(a)**
We know that $|\sin^{-1} x| \le \frac{\pi}{2}$
$\therefore \sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \frac{3\pi}{2}$
$\Rightarrow \sin^{-1} x = \sin^{-1} y = \sin^{-1} z = \frac{\pi}{2}$
$\Rightarrow x = y = z = \sin\frac{\pi}{2} = 1$
$\therefore x^{100} + y^{100} + z^{100} - \frac{9}{x^{101} + y^{101} + z^{101}}$
$= 3 - \frac{9}{3} - 0$

234 **(d)**
We have,
$\sin(\sin^{-1} 1/5 + \cos^{-1} x) = 1$
$\Rightarrow \sin^{-1}\frac{1}{5} + \cos^{-1} x = \frac{\pi}{2}$
$\Rightarrow \sin^{-1}\frac{1}{5} = \frac{\pi}{2} - \cos^{-1} x \Rightarrow \sin^{-1}\frac{1}{5} = \sin^{-1} x \Rightarrow x = \frac{1}{5}$

235 **(d)**
$\because f(x) = ax + b$
$\therefore f'(x) = a > 0$
$\Rightarrow f(x)$ is an increasing function.
$\therefore f(-1) = 0$ and $f(1) = 2$
Or $-a + b = 0$
and $a + b = 2$
then, $a = b = 1$
$\Rightarrow f(x) = x + 1$
Now, $\cot[\cot^{-1} 7 + \cot^{-1} 8 + \cot^{-1} 18]$
$= \cot\left\{\tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{8}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$
$= \cot\left\{\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\cdot\frac{1}{8}}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$
$= \cot\left\{\tan^{-1}\left(\frac{15}{35}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$
$= \cot\left\{\tan^{-1}\left(\frac{3}{11}\right) + \tan^{-1}\left(\frac{1}{18}\right)\right\}$
$= \cot\left\{\tan^{-1}\left(\frac{\frac{3}{11}+\frac{1}{18}}{1-\frac{3}{1\cdot 1}\cdot\frac{1}{18}}\right)\right\}$
$= \cot\left\{\tan^{-1}\left(\frac{65}{195}\right)\right\}$
$= \cot\left\{\tan^{-1}\left(\frac{1}{3}\right)\right\}$
$= \cot(\cot^{-1} 3) = 3 = 1 + 2 = f(2)$

236 **(d)**
$\sin^{-1}\frac{2a}{1+a^2} - \cos^{-1}\frac{1-b^2}{1+b^2} = \tan^{-1}\frac{2x}{1-x^2}$
$\Rightarrow 2\tan^{-1} a - 2\tan^{-1} b = 2\tan^{-1} x$
$\Rightarrow \tan^{-1}\frac{a-b}{1+ab} = \tan^{-1} x$
$\Rightarrow x = \frac{a-b}{1+ab}$

237 **(c)**
We have,
$\cot^{-1} 3 + \operatorname{cosec}^{-1}\sqrt{5} = \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{2}$
$= \tan^{-1} 1 = \frac{\pi}{4}$

238 **(c)**
$\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{2a}{1+a^2}\right) + \frac{1}{2}\cos^{-1}\left(\frac{1-a^2}{1+a^2}\right)\right]$
$= \tan\left[\frac{1}{2}\cdot 2\tan^{-1} a + \frac{1}{2}\cdot 2\tan^{-1} a\right]$
$= \tan(2\tan^{-1} a)$

$= \tan\left[\tan^{-1}\left(\frac{2a}{1-a^2}\right)\right]$

$= \frac{2a}{1-a^2}$

239 **(a)**

Given, $\tan^{-1}x + \tan^{-1}y = \frac{\pi}{4}$

$\therefore \quad \tan^{-1}\left(\frac{x+y}{1-xy}\right) = \frac{\pi}{4}$

$\Rightarrow \quad \frac{x+y}{1-xy} = 1$

$\Rightarrow \quad x + y + xy = 1$

240 **(a)**

Let $\cos^{-1}x = \theta$. Then, $x = \cos\theta$

Also, $0 \le x \le 1 \Rightarrow 0 \le \cos\theta \le 1 \Rightarrow 0 \le \theta \le \frac{\pi}{2}$

Now,

$\cos^{-1}(2x^2-1) = \cos^{-1}(2\cos^2\theta - 1)$

$= \cos^{-1}(\cos 2\theta)$

$= 2\theta = 2\cos^{-1}x \quad [\because 0 \le 2\theta \le \pi]$

241 **(a)**

$$\theta = \tan^{-1}\sqrt{\frac{a(a+b+c)}{bc}} + \tan^{-1}\sqrt{\frac{b(a+b+c)}{ca}} + \tan^{-1}\sqrt{\frac{c(a+b+c)}{ab}}$$

Let $s^2 = \frac{a+b+c}{abc}$

Hence, $\theta = \tan^{-1}\sqrt{a^2s^2} + \tan^{-1}\sqrt{b^2s^2} + \tan^{-1}\sqrt{c^2s^2}$

$= \tan^{-1}(as) + \tan^{-1}(bs) + \tan^{-1}(cs)$

$= \tan^{-1}\left[\frac{as+bs+cs-abcs^3}{1-abs^2-acs^2-bcs^2}\right]$

Hence, $\tan\theta = \left[\frac{s[a+b+c)-abcs^2]}{1-(ab+bc+ca)s^2}\right]$

$= \left[\frac{s[(a+b+c)-(a+b+c)]}{1-s^2(ab+bc+ca)}\right] = 0$

242 **(a)**

Given, $\sin^{-1}x + \sin^{-1}y = \frac{\pi}{2}$

$\therefore \frac{\pi}{2} - \cos^{-1}x + \frac{\pi}{2} - \cos^{-1}y = \frac{\pi}{2}$

$\Rightarrow \cos^{-1}x + \cos^{-1}y = \frac{\pi}{2}$

243 **(c)**

$$\tan^{-1}\left(\frac{c_1 - y}{c_1 y + x}\right) + \tan^{-1}\left(\frac{c_2 - c_1}{1 + c_2 c_1}\right) + \tan^{-1}\left(\frac{c_3 - c_2}{1 + c_3 c_2}\right) + \ldots + \tan^{-1}\frac{1}{c_n}$$

$$= \tan^{-1}\left(\frac{\frac{x}{y} - \frac{1}{c_1}}{1 + \frac{x}{y}\cdot\frac{1}{c_1}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_1} - \frac{1}{c_2}}{1 + \frac{1}{c_1 c_2}}\right) + \tan^{-1}\left(\frac{\frac{1}{c_2} - \frac{1}{c_3}}{1 + \frac{1}{c_2 c_3}}\right) + \ldots + \tan^{-1}\frac{1}{c_n}$$

$$= \tan^{-1}\frac{x}{y} - \tan^{-1}\frac{1}{c_1} + \tan^{-1}\frac{1}{c_1} - \tan^{-1}\frac{1}{c_2} + \tan^{-1}\frac{1}{c\_2} - \tan^{-1}\frac{1}{c_3} + \ldots + \tan^{-1}\frac{1}{c_{n-1}} - \tan^{-1}\frac{1}{c_n} + \tan^{-1}\frac{1}{c_n}$$

$= \tan^{-1}\left(\frac{x}{y}\right)$

244 **(d)**

We have,

$\cos\{\tan^{-1}(\tan 2)\}$

$= \cos\{\tan^{-1}(\tan(2-\pi))\} = \cos(2-\pi)$

$= \cos(\pi - 2) = -\cos 2$

245 **(c)**

We have, $\tan^{-1}\frac{x-1}{x+2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$\Rightarrow \tan^{-1}\left[\frac{\frac{x-1}{x+2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x+2}\right)\left(\frac{x+1}{x+2}\right)}\right] = \frac{\pi}{4}$

$\Rightarrow \left[\frac{2x(x+2)}{x^2+4+4x-x^2+1}\right] = \tan\frac{\pi}{4}$

$\Rightarrow \frac{2x(x+2)}{4x+5} = 1$

$\Rightarrow 2x^2 + 4x = 4x + 5$

$\Rightarrow x = \pm\sqrt{\frac{5}{2}}$

246 **(a)**

Given series can be rewritten as

$$\sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{1+r+r^2}\right)$$

Now, $\tan^{-1}\left(\frac{1}{1+r+r^2}\right)$

$= \tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)$

$= \tan^{-1}(r+1) - \tan^{-1}(r)$

$$\therefore \sum_{r=1}^{n}[\tan^{-1}(r+1) - \tan^{-1}r]$$

$= \tan^{-1}(n+1) - \tan^{-1}(1)$

$= \tan^{-1}(n+1) - \frac{\pi}{4}$

$$\Rightarrow \sum_{r=1}^{\infty} \tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$$

247 **(c)**

Here, $x^2 - 2x + 2 = (x-1)^2 + 1 \geq 1$

But $-1 \leq (x^2 - 2x + 2) \leq 1$

Which is possible only when

$x^2 - 2x + 2 = 1$

$\Rightarrow x = 1$

Then, $a(1)^2 + \sin^{-1}(1) + \cos^{-1}(1) = 0$

$\Rightarrow a + \frac{\pi}{2} + 0 = 0$

$\Rightarrow a = -\frac{\pi}{2}$

248 **(c)**

Given that, $\tan^{-1} x - \tan^{-1} y = \tan^{-1} A$

$\Rightarrow \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1} A$

Hence, $A = \frac{x-y}{1+xy}$

249 **(a)**

$\because \tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}}\right) = \frac{\pi}{2}$

$\Rightarrow \frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}} = \tan\frac{\pi}{2} \Rightarrow 1 - \frac{ab}{x^2} = 0$

$\Rightarrow x^2 = ab \Rightarrow x = \sqrt{ab}$

250 **(d)**

$\cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2} = \frac{\pi}{3} + 2.\frac{\pi}{6} = \frac{2\pi}{3}$

251 **(d)**

We have,

$4\tan^{-1}\frac{1}{5} - \tan^{-1}\frac{1}{239}$

$= 2\tan^{-1}\left(\frac{2/5}{1-1/25}\right) - \tan^{-1}\frac{1}{239}$

$= 2\tan^{-1}(5/12) - \tan^{-1} 1/239$

$= \tan^{-1}\left(\frac{2(2/12)}{(1-5/12)^2}\right) - \tan^{-1}\frac{1}{239}$

$= \tan^{-1}\frac{120}{119} - \tan^{-1}\frac{1}{239}$

$= \tan^{-1}\left(\frac{120/119 - 1/239}{1 + 120/119 \times 1/239}\right)$

$= \tan^{-1}\left(\frac{28569}{28569}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

252 **(c)**

Since, $2\sin^{-1} = \sin^{-1}(2x\sqrt{1-x^2})$

Range of right hand side is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\Rightarrow -\frac{\pi}{2} \leq 2\sin^{-1} x \leq \frac{\pi}{2}$

$\Rightarrow \frac{\pi}{4} \leq \sin^{-1} x \leq \frac{\pi}{4}$

$\Rightarrow x \in \left[-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right]$

254 **(b)**

Sum of two given angles is

$= \cot^{-1} 2 + \cot^{-1} 3$

$= \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \tan^{-1}(1) = \frac{\pi}{4}$

So, the third angle is $\pi - \frac{\pi}{4} = \frac{3\pi}{4}$

255 **(a)**

Roots of equation $x^2 - 9x + 8 = 0$ are 1 and 8

Let $y = [\sin^2\alpha + \sin^4\alpha + \sin^6\alpha + \ldots \infty] \log_e 2$

$\Rightarrow y = \frac{\sin^2\alpha}{1-\sin^2\alpha}\log_e 2 = \tan^2\alpha \log_e 2$

$\Rightarrow y = \log_e 2^{\tan^2\alpha}$

$\Rightarrow e^y = 2^{\tan^2\alpha}$

According to question,

$2^{\tan^2\alpha} = 8 = 2^3 \Rightarrow \tan^2\alpha = 3$

$\Rightarrow \tan\alpha = \sqrt{3} \Rightarrow \alpha = \frac{\pi}{3}$

$\therefore \sin^{-1}\left(\sin\frac{2\pi}{3}\right) = \pi - \frac{2\pi}{3} = \frac{\pi}{3} = \alpha$

256 **(a)**

Let $\cos^{-1} x = \theta$. Then, $x = \cos\theta$

Also, $\frac{1}{2} \leq x \leq 1 \Rightarrow \frac{1}{2} \leq \cos\theta \leq 1 \Rightarrow 0 \leq \theta \leq \frac{\pi}{3}$

Now,

$\cos^{-1}(4x^3 - 3x) = \cos^{-1}(\cos 3\theta)$

$= 3\theta = 3\cos^{-1} x \quad \left[\begin{array}{l}\because 0 \leq \theta \leq \frac{\pi}{3} \\ \Rightarrow 0 \leq 3\theta \leq \pi\end{array}\right]$

257 **(b)**

Let $\sin^{-1} x = \theta$. Then, $x = \sin\theta$ and $\sqrt{1-x^2} = \cos\theta$

Now,

$\sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$= \sin^{-1}(\sin 2\theta) = 2\theta$, if $-\frac{\pi}{2} \leq 2\theta \leq \frac{\pi}{2}$

$= 2\sin^{-1} x$, if $-\frac{\pi}{4} \leq \theta \leq \frac{\pi}{4}$ i.e. if $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

$\therefore \sin^{-1}(2x\sqrt{1-x^2}) - 2\sin^{-1} x = 0$, if $-\frac{1}{\sqrt{2}} \leq x \leq \frac{1}{\sqrt{2}}$

258 **(c)**

$\because [\sin^{-1} x] > [\cos^{-1} x]$

$\Rightarrow x > 0$

Here, $[\cos^{-1} x] = \begin{cases} 0, & x \in (\cos 1, 1) \\ 1, & x \in (0, \cos 1) \end{cases}$

and, $[\sin^{-1} x] = \begin{cases} 0, & x \in (0, \sin 1) \\ 1, & x \in (\sin 1, 1) \end{cases}$

$\therefore x \in [\sin 1, 1)$

$\because \left[\frac{x}{2}\right] = 1$

Or we say that $x \in [\sin 1, 1]$

259 **(a)**

We have, $1 \le \sin^{-1}\cos^{-1}\sin^{-1}\tan^{-1} x \le \frac{\pi}{2}$

$\Rightarrow \sin 1 \le \cos^{-1}\sin^{-1}\tan^{-1} x \le 1$

$\Rightarrow \cos\sin 1 \ge \sin^{-1}\tan^{-1} x \ge \cos 1$

$\Rightarrow \sin\cos\sin 1 \ge \tan^{-1} x \ge \sin\cos 1$

$\Rightarrow \tan\sin\cos\sin 1 \ge x \ge \tan\sin\cos 1$

$\therefore x \in [\tan\sin\cos 1, \tan\sin\cos\sin 1]$

260 **(d)**

Given, $\tan^{-1} x + 2\cot^{-1} x = \frac{2\pi}{3}$

$\therefore \tan^{-1} x + 2\tan^{-1}\frac{1}{x} = \frac{2\pi}{3}$

$\Rightarrow \tan^{-1} x + \tan^{-1}\left(\frac{2\left(\frac{1}{x}\right)}{1-\left(\frac{1}{x}\right)^2}\right) = \frac{2\pi}{3}$

$\Rightarrow \tan^{-1} x + \tan^{-1}\left(\frac{2x}{x^2-1}\right) = \frac{2\pi}{3}$

$\Rightarrow \tan^{-1}\left(\frac{x+\frac{2x}{x^2-1}}{1-\frac{2x^2}{x^2-1}}\right) = \frac{2\pi}{3}$

$\Rightarrow \frac{x(x^2+1)}{-1(x^2+1)} = -\sqrt{3}$

$\Rightarrow x = \sqrt{3}$

261 **(d)**

$\tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{3\sin 2x}{5+3\cos 2x}\right)$

$= \tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{\frac{6\tan x}{1+\tan^2 x}}{5+\frac{3(1-\tan^2 x)}{1+\tan^2 x}}\right)$

$= \tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{6\tan x}{8+2\tan^2 x}\right)$

$= \tan^{-1}\left(\frac{\tan x}{4}\right) + \tan^{-1}\left(\frac{3\tan x}{4+\tan^2 x}\right)$

$= \tan^{-1}\left(\frac{\frac{\tan x}{4}+\frac{3\tan x}{4+\tan^2 x}}{1-\frac{3\tan^2 x}{4(4+\tan^2 x)}}\right)\left(\text{as} \left|\frac{\tan x}{4}\cdot\frac{3\tan x}{4\tan^2 x}\right| < 1\right)$

$= \tan^{-1}\left(\frac{16\tan x + \tan^3 x}{16+\tan^2 x}\right)$

$= \tan^{-1}(\tan x) = x$

262 **(a)**

Let $\sin^{-1} x = \theta$. Then, $x = \sin\theta$

Also,

$-\frac{1}{2} \le x \le \frac{1}{2} \Rightarrow -\frac{1}{2} \le \sin\theta \le \frac{1}{2} \Rightarrow -\frac{\pi}{6} \le \theta \le \frac{\pi}{6}$

Now,

$\sin^{-1}(3x-4x^3)$

$= \sin^{-1}(\sin 3\theta)$

$= 3\theta \quad \left[\because -\frac{\pi}{6} \le \theta \le \frac{\pi}{6} \Rightarrow -\frac{\pi}{2} \le 3\theta \le \frac{\pi}{2}\right]$

$= 3\sin^{-1} x$

263 **(c)**

We have,

$\tan\theta + \tan\left(\frac{\pi}{3}+\theta\right) + \tan\left(\frac{-\pi}{3}+\theta\right) = K\tan 3\theta$

$\Rightarrow \tan\theta + \tan(60+\theta) + \tan(-60+\theta)$

$= K\tan 3\theta$

$\Rightarrow \tan\theta + \frac{\sqrt{3}+\tan\theta}{1-\sqrt{3}\tan\theta} + \frac{\tan\theta-\sqrt{3}}{1+\sqrt{3}\tan\theta} = K\tan 3\theta$

$\Rightarrow \tan\theta + \frac{8\tan\theta}{1-3\tan^2\theta} = K\tan 3\theta$

$\Rightarrow \frac{3(3\tan\theta-\tan^3\theta)}{(1-3\tan^2\theta)} = K\tan 3\theta$

$\Rightarrow 3\tan 3\theta = K\tan 3\theta \Rightarrow K = 3$

264 **(d)**

Let $\cos^{-1} x = \theta$. Then, $x = \cos\theta$

Also,

$-1 \le x \le 0 \Rightarrow -1 \le \cos\theta \le 0 \Rightarrow \frac{\pi}{2} \le \theta \le \pi$

Now,

$\cos^{-1}(2x^2-1) = \cos^{-1}(\cos 2\theta)$

$= \cos^{-1}(2\pi - 2\theta)$

$\Rightarrow \cos^{-1}(2x^2-1)$

$= 2\pi - 2\theta \left[\because \frac{\pi}{2} \le \theta \le \pi \Rightarrow \pi \le 2\theta \le 2\pi \Rightarrow 0 \le 2\pi - 2\theta \le \pi\right]$

$\Rightarrow \cos^{-1}(2x^2-1) = 2\pi - 2\cos^{-1} x$

265 **(c)**

We have,

$\alpha + \beta = \pi$

Also,

$\alpha = \sin^{-1}\frac{\sqrt{3}}{2} + \sin^{-1}\frac{1}{3}$

$\Rightarrow \alpha = \frac{\pi}{3} + \sin^{-1}\frac{1}{3}$

$\Rightarrow \alpha < \frac{\pi}{3} + \sin^{-1}\frac{1}{2}$ [$\because \sin^{-1} x$ is increasing on $[-1, 1]$]

$\Rightarrow \alpha < \frac{\pi}{3} + \frac{\pi}{6} = \frac{\pi}{2}$

$\therefore \alpha + \beta = \pi \Rightarrow \beta > \frac{\pi}{2}$. Thus, $\alpha < \beta$

266 **(a)**

Let $\tan^{-1} x = \theta$. Then, $x = \tan\theta$

Also,

$-1 \le x \le 1 \Rightarrow -1 \le \tan\theta \le 1 \Rightarrow -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

$\Rightarrow -\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}$

Now,

$\sin^{-1}\left(\frac{2x}{1+x^2}\right)$

$= \sin^{-1}(\sin 2\theta)$

$= 2\theta \quad \left[\because -\frac{\pi}{4} \le \theta \le \frac{\pi}{4} \Rightarrow -\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}\right]$

$= 2\tan^{-1} x$

267 **(a)**

$\sin\left[3\sin^{-1}\left(\frac{1}{5}\right)\right] = \sin\left[\sin^{-1}\left\{3\left(\frac{1}{5}\right) - 4\left(\frac{1}{5}\right)^3\right\}\right]$

$= \frac{3}{5} - \frac{4}{125} = \frac{71}{125}$

268 **(a)**

Since, $-\frac{\pi}{2} < \sin^{-1} x \le \frac{\pi}{2}$

$\therefore \sin^{-1} x_i = \frac{\pi}{2}, 1 \le i \le 20$

$\Rightarrow x_i = 1, 1 \le i \le 20$

Thus, $\sum_{i=1}^{20} x_i = 20$

269 **(d)**

Given, $\sin[\cot^{-1}(1+x)] = \cos(\tan^{-1} x)$

$\therefore \sin\left(\sin^{-1}\frac{1}{\sqrt{1+(1+x^2)}}\right)$

$= \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)$

$\Rightarrow \frac{1}{\sqrt{1+(1+x^2)}} = \frac{1}{\sqrt{1+x^2}}$

$\Rightarrow 1 + x^2 + 2x + 1 = x^2 + 1$

$\Rightarrow \quad x = -\frac{1}{2}$

270 **(b)**

$\therefore \tan\left[\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right] + \tan\left[\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]$

$= \tan\left[\frac{\pi}{4} + \phi\right] + \tan\left[\frac{\pi}{4} - \phi\right]$

$\left[\text{put} \frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right) = \phi \Rightarrow \cos 2\phi = \frac{a}{b}\right]$

$= \frac{1+\tan\phi}{1-\tan\phi} + \frac{1-\tan\phi}{1+\tan\phi}$

$= \frac{2(1+\tan^2\phi)}{1-\tan^2\phi}$

$= \frac{2}{\cos 2\phi} = \frac{2b}{a}$

271 **(b)**

$\tan^{-1}\frac{x}{y} - \tan^{-1}\frac{x-y}{x+y}$

$= \tan^{-1}\frac{x}{y} - \tan^{-1}\left[\frac{1-\frac{y}{x}}{1+\frac{y}{x}}\right]$

$= \tan^{-1}\frac{x}{y} - \tan^{-1} 1 + \tan^{-1}\frac{y}{x}$

$= \tan^{-1}\frac{x}{y} + \cot^{-1}\frac{x}{y} - \tan^{-1} 1$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

272 **(c)**

Here, $x^2 - 2x + 2 = (x-1)^2 + 1 \ge 1$

But $-1 \le (x^2 - 2x + 2) \le 1$

Which is possible only when

$x^2 - 2x + 2 = 1$

$\Rightarrow x = 1$

Then, $a(1)^2 + \sin^{-1}(1) + \cos^{-1}(1) = 0$

$\Rightarrow a + \frac{\pi}{2} + 0 = 0$

$\Rightarrow a = -\frac{\pi}{2}$

273 **(d)**

$\cos^{-1}\left(-\frac{1}{2}\right) - 2\sin^{-1}\left(\frac{1}{2}\right)$

$+ 3\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right) + 4\tan^{-1}(-1)$

$= \pi - \cos^{-1}\left(\frac{1}{2}\right) - 2\left(\frac{\pi}{6}\right) + 3\left(\pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right)$

$+4\tan^{-1}(1)$

$= \pi - \frac{\pi}{3} - \frac{\pi}{3} + 3\left(\pi - \frac{\pi}{4}\right) + 4.\frac{\pi}{4}$

$= \frac{\pi}{3} + 3.\frac{3\pi}{4} + \pi = \frac{43\pi}{12}$

274 **(b)**

$\theta = \sin^{-1} x + \cos^{-1} x - \tan^{-1} x = \frac{\pi}{2} - \tan^{-1} x$

$\left[\because \sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}\right]$

$\Rightarrow \quad \theta = \cot^{-1} x$

Since, $1 \le x < \infty$, therefore $0 \le \theta \le \frac{\pi}{4}$

275 **(b)**

Given, $4\sin^{-1} x + \cos^{-1} x = \pi$

$\Rightarrow 4\sin^{-1} x + \frac{\pi}{2} - \sin^{-1} x = \pi$

$\Rightarrow \quad 3\sin^{-1} x = \frac{\pi}{2}$

$\Rightarrow \quad \sin^{-1} x = \frac{\pi}{6}$

$\Rightarrow \quad x = \frac{1}{2}$

276 **(d)**

$\cos\left(\frac{33\pi}{5}\right) = \cos\left(6\pi + \frac{3\pi}{5}\right) = \cos\frac{3\pi}{5}$

$= \sin\left(\frac{\pi}{2} - \frac{3\pi}{5}\right) = \sin\left(-\frac{\pi}{10}\right)$

$= \sin^{-1}\sin\left(-\frac{\pi}{10}\right) = -\frac{\pi}{10}$

277 **(b)**

Given expression

$= \tan\left[\tan^{-1}\frac{a_2 - a_1}{1 + a_1 a_2}\right.$

$\left. + \tan^{-1}\frac{a_3 - a_2}{1 + a_2 a_3} + \ldots + \tan^{-1}\frac{a_n - a_{n-1}}{1 + a_n a_{n-1}}\right]$

$= \tan[\tan^{-1} a_2 - \tan^{-1} a_1 + \tan^{-1} a_3 - \tan^{-1} a_2 + \ldots + \tan^{-1} a_n - \tan^{-1} a_{n-1}]$

$= \tan[\tan^{-1} a_n - \tan^{-1} a_1] = \frac{a_n - a_1}{1 + a_1 a_n}$

$= \frac{(n-1)d}{1 + a_1 a_n}$

278 **(a)**

$\because \tan^{-1}\left(\frac{a}{x}\right) + \tan^{-1}\left(\frac{b}{x}\right) = \frac{\pi}{2}$

$\Rightarrow \tan^{-1}\left(\frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}}\right) = \frac{\pi}{2}$

$\Rightarrow \frac{\frac{a}{x}+\frac{b}{x}}{1-\frac{ab}{x^2}} = \tan\frac{\pi}{2} \Rightarrow 1 - \frac{ab}{x^2} = 0$

$\Rightarrow x^2 = ab \Rightarrow x = \sqrt{ab}$

280 **(b)**

Given, $\tan\left\{\sec^{-1}\left(\frac{1}{x}\right)\right\} = \sin(\tan^{-1} 2)$

$\Rightarrow \tan\left(\tan^{-1}\frac{\sqrt{1-x^2}}{x}\right) = \sin\left(\sin^{-1}\frac{2}{\sqrt{1+2^2}}\right)$

$\left[\because \tan^{-1} x = \sin^{-1}\frac{x}{\sqrt{1+x^2}}\right]$

$\Rightarrow \frac{\sqrt{1-x^2}}{x} = \frac{2}{\sqrt{5}}$

$\Rightarrow 4x^2 = 5(1-x^2)$

$\Rightarrow x^2 = \frac{5}{9} \Rightarrow x = \frac{\sqrt{5}}{3}$

282 **(b)**

Given, $(\sqrt{3} - i) = (a + ib)(c + id)$

$= (ac - bd) + i(ad + bc)$

On comparing the real and imaginary part on both sides, we get

$ac - bd = \sqrt{3}$

And $ad + bc = 1$

Now, $\tan^{-1}\left(\frac{b}{a}\right) + \tan^{-1}\left(\frac{d}{c}\right)$

$= \tan^{-1}\left(\frac{bc + ad}{ac - bd}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= n\pi + \frac{\pi}{6}, n \in I$

283 **(b)**

Given, $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1} x$

Let $x = \tan\theta$

$\therefore \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \frac{1}{2}\tan^{-1}(\tan\theta)$

$\Rightarrow \tan^{-1}\left\{\tan\left(\frac{\pi}{4} - \theta\right)\right\} = \frac{1}{2}\tan^{-1}(\tan\theta)$

$\Rightarrow \frac{\pi}{4} - \theta = \frac{\theta}{2} \Rightarrow \theta = \frac{\pi}{6}$

$\therefore x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

284 **(c)**

Let $S_\infty = \cot^{-1} 2 + \cot^{-1} 8 + \cot^{-1} 18 + \cot^{-1} 32 + \ldots$

$\therefore T_n \cot^{-1} 2n^2$

$= \tan^{-1}\frac{1}{2n^2}$

$= \tan^{-1}\left(\frac{2}{4n^2}\right) = \tan^{-1}\left(\frac{(2n+1)-(2n-1)}{1+(2n+1)(2n-1)}\right)$

$\therefore S_n = \sum_{n=1}^{\infty}\{\tan^{-1}(2n+1) - \tan^{-1}(2n-1)\}$

$= \tan^{-1}\infty - \tan^{-1} 1$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

286 **(d)**

$\sin^{-1}\left\{\tan\left(\frac{-5\pi}{4}\right)\right\} = \sin^{-1}\left\{-\tan\left(\pi + \frac{\pi}{4}\right)\right\}$

$= \sin^{-1}\left(-\tan\frac{\pi}{4}\right)$

$= \sin^{-1}\left(-\sin\frac{\pi}{2}\right)$

$= -\frac{\pi}{2}$

287 **(a)**

$\because \tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}\left(\frac{r+1-r}{1+r(r+1)}\right)$

$= \tan^{-1}(r+1) - \tan^{-1}(r)$

$\therefore \sum_{r=0}^{n}[\tan^{-1}(r+1) - \tan^{-1}(r)]$

$= \tan^{-1}(n+1) - \tan^{-1}(0)$

$= \tan^{-1}(n+1)$

$\Rightarrow \sum_{r=0}^{\infty}\tan^{-1}\left(\frac{1}{1+r+r^2}\right) = \tan^{-1}(\infty) = \frac{\pi}{2}$

288 **(c)**

Let $\cos^{-1} x = \theta$. Then, $x = \cos\theta$

Also,

$-1 \le x \le -\frac{1}{2} \Rightarrow -1 \le \cos\theta \le -\frac{1}{2} \Rightarrow \frac{2\pi}{3} \le \theta \le \pi$

Now,

$\cos^{-1}(4x^3 - 3x)$

$= \cos^{-1}(\cos 3\theta)$

$= \cos^{-1}(\cos(2\pi - 3\theta))$

$= \cos^{-1}(\cos(3\theta - 2\pi))$

$= 3\theta - 2\pi \quad \left[\because \frac{2\pi}{3} \le \theta \le \pi \Rightarrow 0 \le 3\theta - 2\pi \le \pi\right]$

$= 3\cos^{-1} x - 2\pi$

289 **(c)**

Given that, $\tan^{-1}x - \tan^{-1}y = \tan^{-1}A$

$\Rightarrow \tan^{-1}\left(\frac{x-y}{1+xy}\right) = \tan^{-1}A$

Hence, $A = \frac{x-y}{1+xy}$

290 **(b)**

We have, $2\tan^{-1}(\cos x) = \tan^{-1}(2\,\mathrm{cosec}\,x)$

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\,\mathrm{cosec}\,x)$

$\Rightarrow \frac{2\cos x}{\sin^2 x} = 2\,\mathrm{cosec}\,x$

$\Rightarrow \sin x = \cos x \Rightarrow x = \frac{\pi}{4}$

291 **(c)**

Clearly, $x(x+1) \geq 0$ and $x^2+x+1 \leq 1$

$\Rightarrow x(x+1) = 0$

$\Rightarrow x = 0, -1$

When $x = 0$,

LHS=$\tan^{-1}0 + \sin^{-1}1 = \frac{\pi}{2}$

When $x = -1$,

LHS=$\tan^{-1}0 + \sin^{-1}\sqrt{1-1+1}$

$= 0 + \sin^{-1}(1) = \frac{\pi}{2}$

Thus, the number of solution is 2

292 **(b)**

We have,

$\cos\left\{\cos^{-1}\left(-\frac{1}{7}\right) + \sin^{-1}\left(-\frac{1}{7}\right)\right\} = \cos\frac{\pi}{2} = 0$

293 **(c)**

The given equation is satisfied only when $x = 1$, $y = -1, z = 1$

294 **(c)**

Given, $\sin^{-1}(1-x) = \frac{\pi}{2} + 2\sin^{-1}x$

$\Rightarrow\ 1 - x = \sin\left(\frac{\pi}{2} + 2\sin^{-1}x\right)$

$\Rightarrow\ 1 - x = \cos(2\sin^{-1}x)$

$\Rightarrow\ 1 - x = \cos(2\cos^{-1}\sqrt{1-x^2})$

$\Rightarrow\ 1 - x = \cos\{\cos^{-1}(1-2x^2)\}$

$\Rightarrow\ 1 - x = 1 - 2x^2$

$\Rightarrow\ x = 0, \frac{1}{2}$

$\Rightarrow\ x$

$= 0\ \left[\because x = \frac{1}{2} \text{ does not satisfy the given equation}\right]$

295 **(d)**

We have,

$\cos^{-1}\left(\frac{15}{17}\right) + 2\tan^{-1}\left(\frac{1}{5}\right)$

$= \cos^{-1}\left(\frac{15}{17}\right) + \cos^{-1}\left(\frac{1-1/25}{1+1/25}\right)$

$= \cos^{-1}\left(\frac{15}{17}\right) + \cos^{-1}\left(\frac{12}{13}\right)$

$= \cos^{-1}\left\{\frac{15}{17}\times\frac{12}{13} - \sqrt{1-\left(\frac{15}{17}\right)^2}\sqrt{1-\left(\frac{12}{13}\right)^2}\right\}$

$= \cos^{-1}\left(\frac{140}{221}\right)$

296 **(c)**

Let $\cot^{-1}x = \theta$. Then, $x = \cot\theta$

Also, $x < 0 \Rightarrow \cot\theta < 0 \Rightarrow \frac{\pi}{2} < \theta < \pi$

Now,

$\tan^{-1}\left(\frac{1}{x}\right)$

$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(\tan\theta)$

$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(-\tan(\pi-\theta))$

$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \tan^{-1}(\tan(\theta-\pi))$

$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \theta$

$-\pi\ \left[\frac{\pi}{2} < \theta < \pi \Rightarrow -\frac{\pi}{2} < \theta - \pi < 0\right]$

$\Rightarrow \tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1}x - \pi$

297 **(d)**

Let $\alpha = \cos^{-1}\sqrt{P}, \beta = \cos^{-1}\sqrt{1-P}$

And $\gamma = \cos^{-1}\sqrt{1-q}$

$\Rightarrow \cos\alpha = \sqrt{p},\ \cos\beta = \sqrt{1-p}$

And $\cos\gamma = \sqrt{1-q}$

Therefore, $\sin\alpha = \sqrt{1-p}, \sin\beta = \sqrt{p}$ and $\sin\gamma = \sqrt{q}$

The given equation may be written as

$\alpha + \beta + \gamma = \frac{3\pi}{4}$

$\Rightarrow \alpha + \beta = \frac{3\pi}{4} - \gamma$

$\Rightarrow \cos(\alpha+\beta) = \cos\left(\frac{3\pi}{4} - \gamma\right)$

$\Rightarrow \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \cos\left\{\pi - \left(\frac{\pi}{4} + \gamma\right)\right\} = -\cos\left(\frac{\pi}{4} + \gamma\right)$

$\Rightarrow \sqrt{p}\sqrt{1-p} - \sqrt{1-p}\sqrt{p}$

$= -\left(\frac{1}{\sqrt{2}}\sqrt{1-q} - \frac{1}{\sqrt{2}}\sqrt{q}\right)$

$\Rightarrow 0 = \sqrt{1-q} - \sqrt{q} \Rightarrow 1 - q = q$

$\Rightarrow q = \frac{1}{2}$

298 **(c)**

$$\tan^{-1} = \frac{m}{n} - \tan^{-1}\frac{m-n}{m+n}$$
$$= \tan^{-1}\frac{m}{n} - \tan^{-1}\frac{\frac{m}{n}-1}{1+\frac{m}{n}}$$
$$= \tan^{-1}\frac{m}{n} - \tan^{-1}\frac{m}{n} + \tan^{-1}(1) = \frac{\pi}{4}$$

299 **(c)**

$$\sin\left[\frac{\pi}{2} - \sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right] = \cos\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$$
$$= \cos\cos^{-1}\sqrt{1-\frac{3}{4}}$$
$$= \cos\cos^{-1}\left(\frac{1}{2}\right) = \frac{1}{2}$$

300 **(d)**

$\cos^{-1} x, \sin^{-1} x$ are real, if $-1 \le x \le 1$

But $\cos^{-1} x > \sin^{-1} x$

$$\Rightarrow 2\cos^{-1} x > \frac{\pi}{2}$$
$$\Rightarrow \cos^{-1} x = \frac{\pi}{4}$$
$$\therefore \cos(\cos^{-1} x) < \cos\frac{\pi}{4}$$
$$\Rightarrow x < \frac{1}{\sqrt{2}}$$

The common value are $-1 \le x < \frac{1}{\sqrt{2}}$

301 **(a)**

$$\cot^{-1}\frac{xy+1}{x-y} + \cot^{-1}\frac{yz+1}{y-z} + \cot^{-1}\frac{zx+1}{z-x}$$
$$= \cot^{-1} y - \cos^{-1} x + \cot^{-1} z - \cot^{-1} y + \cot^{-1} x - \cot^{-1} z$$
$$= 0$$

302 **(a)**

$$\tan\left\{\cos^{-1}\left(-\frac{2}{7}\right) - \frac{\pi}{2}\right\}$$
$$= \tan\left\{\pi - \cos^{-1}\left(\frac{2}{7}\right) - \frac{\pi}{2}\right\}$$
$$= \tan\left\{\frac{\pi}{2} - \cos^{-1}\left(\frac{2}{7}\right)\right\} = \tan\left\{\sin^{-1}\left(\frac{2}{7}\right)\right\}$$
$$= \tan\left\{\tan^{-1}\left(\frac{3}{3\sqrt{5}}\right)\right\} = \frac{2}{3\sqrt{5}}$$

303 **(a)**

$$\sin^{-1} x + \sin^{-1}\frac{1}{x} + \cos^{-1} x + \cos^{-1}\frac{1}{x}$$
$$= [\sin^{-1} x + \cos^{-1} x] + \left[\sin^{-1}\left(\frac{1}{x}\right) + \cos^{-1}\left(\frac{1}{x}\right)\right]$$
$$= \frac{\pi}{2} + \frac{\pi}{2} = \pi$$

304 **(b)**

We know that

$$2\tan^{-1} x = \pi + \tan^{-1}\left(\frac{2x}{1-x^2}\right), \text{if } x > 1$$
$$\therefore x = \sin(2\tan^{-1} 2)$$
$$\Rightarrow x = \sin\left\{\pi + \tan^{-1}\left(\frac{4}{1-4}\right)\right\}$$
$$\Rightarrow x = \sin\left(\pi - \tan^{-1}\frac{4}{3}\right) = \sin\left(\tan^{-1}\frac{4}{3}\right)$$
$$= \sin\left(\sin^{-1}\frac{4}{5}\right) = \frac{4}{5}$$

And,

$$y = \sin\left(\frac{1}{2}\tan^{-1}\frac{4}{5}\right)$$
$$\Rightarrow y = \sin\frac{\theta}{2}, \text{where } \theta = \tan^{-1}\frac{4}{3}\ i.e. \tan\theta = \frac{4}{3}$$
$$\Rightarrow y = \sqrt{\frac{1-\cos\theta}{2}} = \sqrt{\frac{1-3/5}{2}} = \frac{1}{\sqrt{5}}$$

Clearly, $x = 1 - y^2$ or, $y^2 = 1 - x$

306 **(c)**

Given, $\tan^{-1}\sqrt{x(x+1)} = \frac{\pi}{2} - \sin^{-1}\sqrt{x^2+x+1}$

$$\Rightarrow \cos^{-1}\frac{1}{\sqrt{(x^2+x)^2+1}} = \cos^{-1}\sqrt{x^2+x+1}$$
$$\Rightarrow \frac{1}{\sqrt{(x^2+x)^2+1}} = \sqrt{x^2+x+1}$$
$$\Rightarrow 1 = (x^2+x+1)[(x^2+x)^2+1]$$
$$\Rightarrow (x^2+x)^3 + (x^2+x)^2 + (x^2+x) + 1 = 1$$
$$\Rightarrow (x^2+x)\{(x^2+x)^2 + (x^2+x) + 1\} = 0$$
$$\Rightarrow x^2 + x = 0$$
$$\Rightarrow x = 0, -1$$

307 **(b)**

Let $\tan^{-1} x = \theta$. Then, $x = \tan\theta$

Also,

$$-\infty < x \le 0 \Rightarrow -\infty < \tan\theta \le 0 \Rightarrow -\frac{\pi}{2} < \theta \le 0$$
$$\Rightarrow -\pi < 2\theta \le 0$$

Now,

$$\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$$
$$= \cos^{-1}(\cos 2\theta)$$
$$= \cos^{-1}(\cos(-2\theta))$$
$$= -2\theta = -2\tan^{-1} x \quad [\because 0 \le -2\theta < \pi]$$

308 **(c)**

$$\tan(\sin^{-1} x) = \tan\left(\tan^{-1}\frac{x}{\sqrt{1-x^2}}\right), x \in (-1, 1)$$
$$= \frac{x}{\sqrt{1-x^2}}$$